```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index);
> read "EFE" : read "SFE" :read "fids" :read "Seneqs1a" : read "deqs1a" :
```

Chapter XX

Tensor analysis using indices - Senovilla et al. - Shearfree for dust

if $\sigma_{a\,b}$=0 =>ωΘ=0

Author: Peter Huf

file 1b: equations 7-10

The expansion tensor is defined as:

****

Equation 7 Definition of the expansion scalar

****

```
> eq[7] := theta = u[a,-A] : T(%);
```

$$\theta = u^{a}{}_{;a} \tag{1.1}$$

****

Equation 8 Definition of the shear tensor

****

The shear tensor is defined as:

```
> eq[8] := sigma[-a,-b] = (1/2)·P[-a, c]·P[-b, d]·u[-c,-D] + (1/2)·P[-b, c]·P[-a, d]·u[-c,-D] - (1/3)·theta·P[-a,-b] : T(%);
```

$$\sigma_{a\,b} = \frac{1}{2} P_{a}{}^{c} P_{b}{}^{d} u_{c\,;d} + \frac{1}{2} P_{b}{}^{c} P_{a}{}^{d} u_{c\,;d} - \frac{1}{3}\,\theta\, P_{a\,b} \tag{1.2}$$

****

Equations 8abc Properties of the shear tensor

****

We show the following identities of the shear tensor:

```
> deq[8 b] := contract(raise(eq[8], a), a, b) : T(%);
```

$$\sigma^{a}{}_{a} = \frac{1}{2} P^{a\,c} P_{a}{}^{d} u_{c\,;d} + \frac{1}{2} P_{a}{}^{c} P^{a\,d} u_{c\,;d} - \frac{1}{3}\,\theta\, P^{a}{}_{a} \tag{1.3}$$

using the identity:

```
> id[3] := u[-a,-B]·u[a] = 0 : T(%);
```

(1.4)

$$u_{a;b}\,u^{a}=0 \tag{1.4}$$

> deq[1 b] : T(%);

$$P^{a}{}_{b}=u^{a}\,u_{b}+g^{a}{}_{b} \tag{1.5}$$

> deq[1 b1] := raise(deq[1], b) : T(%);

$$P_{a}{}^{b}=u^{b}\,u_{a}+g_{a}{}^{b} \tag{1.6}$$

> deq[1 e] : T(%);

$$P^{b}{}_{b}=3 \tag{1.7}$$

> deq[8 b1] := deq[8 b] : T(%);

$$\sigma^{a}{}_{a}=\frac{1}{2}\,P^{ac}P_{a}{}^{d}u_{c;d}+\frac{1}{2}\,P_{a}{}^{c}P^{ad}u_{c;d}-\frac{1}{3}\,\theta\,P^{a}{}_{a} \tag{1.8}$$

> deq[8 b1] := (TEDS(deq[1 e], deq[8 b1])) : T(%);

$$\sigma^{a}{}_{a}=\frac{1}{2}\,P^{ac}P_{a}{}^{d}u_{c;d}+\frac{1}{2}\,P_{a}{}^{c}P^{ad}u_{c;d}-\frac{1}{3}\,\theta\,P^{a}{}_{a} \tag{1.9}$$

> deq[8 b1] := (TEDS(deq[1 b1], deq[8 b1])) : T(%);

$$\sigma^{a}{}_{a}=\frac{1}{2}\,P^{ac}P_{a}{}^{d}u_{c;d}+\frac{1}{2}\,P_{a}{}^{c}P^{ad}u_{c;d}-\frac{1}{3}\,\theta\,P^{a}{}_{a} \tag{1.10}$$

> deq[1 b1] := subs(b = c, raise(raise(deq[1], a), b)) : T(%);

$$P^{ac}=u^{a}\,u^{c}+g^{ac} \tag{1.11}$$

>

> deq[8 b1] := TEDS(deq[1 b1], deq[8 b1]) : T(%);

$$\sigma^{a}{}_{a}=\frac{1}{2}\,P_{a}{}^{d}u^{a}u^{c}u_{c;d}+\frac{1}{2}\,P_{a}{}^{d}g^{ac}u_{c;d}+\frac{1}{2}\,P_{a}{}^{c}P^{ad}u_{c;d}-\frac{1}{3}\,\theta\,P^{a}{}_{a} \tag{1.12}$$

> deq[1 b2] := subs(b = d, raise(deq[1], b)) : T(%);

$$P_{a}{}^{d}=u^{d}\,u_{a}+g_{a}{}^{d} \tag{1.13}$$

> deq[8 b1] := TEDS(deq[1 b2], deq[8 b1]) : T(%);

$$\sigma^{a}{}_{a}=\frac{1}{2}\,u^{a}u^{c}u^{d}u_{a}u_{c;d}+\frac{1}{2}\,g_{a}{}^{d}u^{a}u^{c}u_{c;d}+\frac{1}{2}\,g^{ac}u^{d}u_{a}u_{c;d}+\frac{1}{2}\,g^{ac}g_{a}{}^{d}u_{c;d}+\frac{1}{2}\,P_{a}{}^{c}P^{ad}u_{c;d}-\frac{1}{3}\,\theta\,P^{a}{}_{a} \tag{1.14}$$

> deq[8 b2] := Absorbg(deq[8 b1]) : T(%);

$$\sigma^{a}{}_{a}=\frac{1}{2}\,u^{a}u^{c}u^{d}u_{a}u_{c;d}+u^{d}u^{c}u_{c;d}+\frac{1}{2}\,g^{cd}u_{c;d}+\frac{1}{2}\,P_{a}{}^{c}P^{ad}u_{c;d}-\frac{1}{3}\,\theta\,P^{a}{}_{a} \tag{1.15}$$

> deq[8 b3] := Absorbg(deq[8 b2]) : T(%);

$$\sigma^{a}{}_{a} = \frac{1}{2} u^{a} u^{c} u^{d} u_{a} u_{c;d} + u^{d} u^{c} u_{c;d} + \frac{1}{2} u^{d}{}_{;d} + \frac{1}{2} P_{a}{}^{c} P^{a d} u_{c;d} - \frac{1}{3} \theta P^{a}{}_{a} \quad (1.16)$$

> deq[1 b3] := subs(b = c, raise(deq[1], b)) : T(%);

$$P_{a}{}^{c} = u^{c} u_{a} + g_{a}{}^{c} \quad (1.17)$$

> deq[8 b4] := TEDS(deq[1 b3], deq[8 b3]) : T(%);

$$\sigma^{a}{}_{a} = \frac{1}{2} u^{a} u^{c} u^{d} u_{a} u_{c;d} + u^{d} u^{c} u_{c;d} + \frac{1}{2} u^{d}{}_{;d} + \frac{1}{2} P^{a d} u^{c} u_{a} u_{c;d} + \frac{1}{2} P^{a d} g_{a}{}^{c} u_{c;d} - \frac{1}{3} \theta P^{a}{}_{a} \quad (1.18)$$

>

> deq[1 b4] := subs(c = d, raise(deq[1 b3], a)) : T(%);

$$P^{a d} = u^{a} u^{d} + g^{a d} \quad (1.19)$$

> deq[8 b5] := TEDS(deq[1 b4], deq[8 b4]) : T(%);

$$\sigma^{a}{}_{a} = u^{a} u^{c} u^{d} u_{a} u_{c;d} + u^{d} u^{c} u_{c;d} + \frac{1}{2} u^{d}{}_{;d} + \frac{1}{2} g^{a d} u^{c} u_{a} u_{c;d} + \frac{1}{2} g_{a}{}^{c} u^{a} u^{d} u_{c;d} + \frac{1}{2} g^{a d} g_{a}{}^{c} u_{c;d} - \frac{1}{3} \theta P^{a}{}_{a} \quad (1.20)$$

> deq[8 b6] := Absorbg(deq[8 b5]) : T(%);

$$\sigma^{a}{}_{a} = u^{a} u^{c} u^{d} u_{a} u_{c;d} + 2 u^{d} u^{c} u_{c;d} + \frac{1}{2} u^{d}{}_{;d} + \frac{1}{2} g^{d c} u_{c;d} - \frac{1}{3} \theta P^{a}{}_{a} \quad (1.21)$$

> deq[8 b7] := Absorbg(deq[8 b6]) : T(%);

$$\sigma^{a}{}_{a} = u^{a} u^{c} u^{d} u_{a} u_{c;d} + 2 u^{d} u^{c} u_{c;d} + \frac{1}{2} u^{d}{}_{;d} + \frac{1}{2} u_{c}{}^{;c} - \frac{1}{3} \theta P^{a}{}_{a} \quad (1.22)$$

> id[3 a] := subs(a = c, -B = -D, id[3]) : T(%);

$$u_{c;d} u^{c} = 0 \quad (1.23)$$

> deq[8 b8] := TEDS(id[3 a], deq[8 b7]) : T(%);

$$\sigma^{a}{}_{a} = \frac{1}{2} u^{d}{}_{;d} + \frac{1}{2} u_{c}{}^{;c} - \frac{1}{3} \theta P^{a}{}_{a} \quad (1.24)$$

> eq[7 a] := subs(a = d, A = D, eq[7]) : T(%);

$$\theta = u^{d}{}_{;d} \quad (1.25)$$

> deq[8 b9] := TEDS(eq[7 a], deq[8 b8]) : T(%);

$$\sigma^{a}{}_{a} = \frac{1}{2} u^{d}{}_{;d} + \frac{1}{2} u_{c}{}^{;c} - \frac{1}{3} P^{a}{}_{a} u^{d}{}_{;d} \quad (1.26)$$

> deq[8 b10] := TEDS(P[a, -a] = 3, deq[8 b9]) : T(%);

$$\sigma^{a}{}_{a} = -\frac{1}{2} u^{d}{}_{;d} + \frac{1}{2} u_{c}{}^{;c} \quad (1.27)$$

>

So we can say

```
> deq[8 b] := sigma[a,-a] = 0 : T(%);
```

$$\sigma^{a}{}_{a} = 0 \quad (1.28)$$

We also want to show that;

```
> deq[8 c] := sigma[-a,-b]·u[b] = 0 : T(%);
```

$$\sigma_{a b} u^{b} = 0 \quad (1.29)$$

```
> eq[8] : T(%);
```

$$\sigma_{a b} = \frac{1}{2} P_{a}{}^{c} P_{b}{}^{d} u_{c;d} + \frac{1}{2} P_{b}{}^{c} P_{a}{}^{d} u_{c;d} - \frac{1}{3} \theta P_{a b} \quad (1.30)$$

```
> deq[8 c1] := sigma[-a,-b]·u[b] : T(%);
```

$$\sigma_{a b} u^{b} \quad (1.31)$$

```
> deq[8 c2] := expand(TEDS(eq[8], deq[8 c1])) : T(%);
```

$$\frac{1}{2} P_{a}{}^{c} P_{b}{}^{d} u^{b} u_{c;d} + \frac{1}{2} P_{a}{}^{d} P_{b}{}^{c} u^{b} u_{c;d} - \frac{1}{3} \theta P_{a b} u^{b} \quad (1.32)$$

```
> deq[3 c1] := raise(deq[3 c], a) : T(%);
```

$$P_{a b} u^{b} = 0 \quad (1.33)$$

```
> deq[8 c3] := expand(TEDS(deq[3 c1], deq[8 c2])) : T(%);
```

$$\frac{1}{2} P_{a}{}^{c} P_{b}{}^{d} u^{b} u_{c;d} + \frac{1}{2} P_{a}{}^{d} P_{b}{}^{c} u^{b} u_{c;d} \quad (1.34)$$

```
> deq[3 c2] := subs(b = e, deq[3 c1]) : T(%);
```

$$P_{a e} u^{e} = 0 \quad (1.35)$$

```
> deq[2 b1] := subs(b = e, deq[2 b]) : T(%);
```

$$P_{a e} = P_{e a} \quad (1.36)$$

```
> deq[3c3] := subs(e = b, a = d, raise(TEDS(deq[2 b1], deq[3 c2]), a)) : T(%);
```

$$u^{b} P_{b}{}^{d} = 0 \quad (1.37)$$

```
> deq[8 c4] := TEDS(deq[3 c3], deq[8 c3]) : T(%);
```

$$\frac{1}{2} P_{a}{}^{d} P_{b}{}^{c} u^{b} u_{c;d} \quad (1.38)$$

```
> deq[3 c4] := subs(d = c, deq[3 c3]) : T(%);
```

$$u^{b} P_{b}{}^{c} = 0 \quad (1.39)$$

```
> deq[8 c5] := TEDS(deq[3 c4], deq[8 c4]) : T(%);
```

$$0 \quad \textbf{(1.40)}$$

and thus the equation is shown.

****

Equation 9 Definition of the vorticity tensor

****

The definition of vorticity tensor is:

> $eq[9] := \text{omega}[-a,-b] = \left(\frac{1}{2}\right)\cdot P[-a,c]\cdot P[-b,d]\cdot u[-c,-\text{D}] - \left(\frac{1}{2}\right)\cdot P[-b,c]\cdot P[-a,d]\cdot u[-c,-\text{D}] : T(\%);$

$$\omega_{a\,b} = \frac{1}{2}\, P_{a}{}^{c} P_{b}{}^{d} u_{c\,;d} - \frac{1}{2}\, P_{b}{}^{c} P_{a}{}^{d} u_{c\,;d} \quad \textbf{(1.41)}$$

****

Equations 9ab Properties of the vorticity tensor

****

>

We can see from observation that omega is antisymmetric:

> $deq[9\,a] := \text{omega}[-a,-b] = -\text{omega}[-b,-a] : T(\%);$

$$\omega_{a\,b} = -\omega_{b\,a} \quad \textbf{(1.42)}$$

We also want to show that :

> $deq[9\,b] := \text{omega}[-a,-b]\cdot u[b] = 0 : T(\%);$

$$\omega_{a\,b} u^{b} = 0 \quad \textbf{(1.43)}$$

> $deq[9\,b1] := \text{omega}[-a,-b]\cdot u[b] : T(\%);$

$$\omega_{a\,b} u^{b} \quad \textbf{(1.44)}$$

> $deq[9\,b2] := expand(TEDS(eq[9], deq[9\,b1])) : T(\%);$

$$\frac{1}{2}\, P_{a}{}^{c} P_{b}{}^{d} u^{b} u_{c\,;d} - \frac{1}{2}\, P_{a}{}^{d} P_{b}{}^{c} u^{b} u_{c\,;d} \quad \textbf{(1.45)}$$

> $deq[3\,c1] := raise(deq[3\,c], a) : T(\%);$

$$P_{a\,b} u^{b} = 0 \quad \textbf{(1.46)}$$

> $deq[8\,c3] := expand(TEDS(deq[3\,c1], deq[8\,c2])) : T(\%);$

$$\frac{1}{2}\, P_{a}{}^{c} P_{b}{}^{d} u^{b} u_{c\,;d} + \frac{1}{2}\, P_{a}{}^{d} P_{b}{}^{c} u^{b} u_{c\,;d} \quad \textbf{(1.47)}$$

> $deq[3\,c2] := subs(b = e, deq[3\,c1]) : T(\%);$

$$P_{a\,e} u^{e} = 0 \quad \textbf{(1.48)}$$

> $deq[2\,b1] := subs(b = e, deq[2\,b]) : T(\%);$

$$P_{a\,e} = P_{e\,a} \quad \textbf{(1.49)}$$

> $deq[3c3] := subs(e = b, a = d, raise(TEDS(deq[2\,b1], deq[3\,c2]), a)) : T(\%);$

$$u^{b} P_{b}^{\ d} = 0 \quad \mathbf{(1.50)}$$

```
> deq[8 c4] := TEDS(deq[3 c3], deq[8 c3]) : T(%);
```

$$\frac{1}{2} P_{a}^{\ d} P_{b}^{\ c} u^{b} u_{c;d} \quad \mathbf{(1.51)}$$

```
> deq[3 c4] := subs(d = c, deq[3 c3]) : T(%);
```

$$u^{b} P_{b}^{\ c} = 0 \quad \mathbf{(1.52)}$$

```
> deq[8 c5] := TEDS(deq[3 c4], deq[8 c4]) : T(%);
```

$$0 \quad \mathbf{(1.53)}$$

and so we have shown:

```
> deq[9 c] := omega[-a,-b]·u[b] = 0 : T(%);
```

$$\omega_{a b} u^{b} = 0 \quad \mathbf{(1.54)}$$

```
> deq[9 a] : T(%);
```

$$\omega_{a b} = -\omega_{b a} \quad \mathbf{(1.55)}$$

```
> deq[9 d] := -1· TEDS(deq[9 a], deq[9 c]) : T(%);
```

$$u^{b} \omega_{b a} = 0 \quad \mathbf{(1.56)}$$

****

Equation 10 Definition of the vorticity vector

****

We introduce the vorticity vector field:

```
> eq[10] := omega[a] = (1/2)·eta[a, b, c, d]·u[-b]·omega[-c,-d] : T(%);
```

$$\omega^{a} = \frac{1}{2} \eta^{a b c d} u_{b} \omega_{c d} \quad \mathbf{(1.57)}$$

****

Equation 10a Property of the vorticity vector

****

It can immediately be shown that

```
> deq[10 a] := omega[-a]·u[a] = 0 : T(%);
```

$$\omega_{a} u^{a} = 0 \quad \mathbf{(1.58)}$$

Proof: since the substitution of eq[10] into deq[10a] gives:

```
> deq[10 a1] := TEDS(raise(eq[10], a), deq[10 a]) : T(%);
```

$$\frac{1}{2} u^{a} \eta_{a}^{\ b c d} u_{b} \omega_{c d} = 0 \quad \mathbf{(1.59)}$$

which contains symmetric ($u^{a} u_{b}$) and antisymmetric $\left(\eta_{a}^{\ bcd}\right)$ products with the same indices

. It is known that such a product is always = 0.

>

It is possible to show that this equation can be inverted to achieve
$\omega_{ab} = \eta_{abef}\,\omega^{e} u^{f}$

We do so as follows. Firstly multiply equation 10 by the velocity u^{e}:

```
> eq[10 a] := lhs(eq[10])·u[e] = rhs(eq[10])·u[e] : T(%);
```

$$\omega^{a} u^{e} = \frac{1}{2}\,\eta^{a\ b\ c\ d}\, u_{b}\,\omega_{cd}\, u^{e} \tag{1.60}$$

Then multiply by η_{fgae}:

```
> eq[10 b] := eta[-f,-g,-a,-e]·lhs(eq[10 a]) = eta[-f,-g,-a,-e]·rhs(eq[10 a]) : T(%);
```

$$\eta_{fgae}\,\omega^{a} u^{e} = \frac{1}{2}\,\eta_{fgae}\,\eta^{a\ b\ c\ d}\, u_{b}\,\omega_{cd}\, u^{e} \tag{1.61}$$

At this point we need to look relook at some critical identities of η^{abcd} and $\delta^{a}{}_{b}$
(It might be better to refer to another file for the following identities:)

>

```
> deltaid[1 a] := delta[a,-b]·u[b] = u[a] : T(%);
```

$$\delta^{a}{}_{b}\, u^{b} = u^{a} \tag{1.62}$$

```
> deltaid[1 b] := delta[a,-b]·u[-a] = u[-b] : T(%);
```

$$\delta^{a}{}_{b}\, u_{a} = u_{b} \tag{1.63}$$

```
> deltaid[1 c] := delta[a,-b]·omega[-a,-c] = omega[-b,-c] : T(%);
```

$$\delta^{a}{}_{b}\,\omega_{ac} = \omega_{bc} \tag{1.64}$$

```
> deltaid[1 d] := delta[a,-b]·omega[-c,-a] = omega[-c,-b] : T(%);
```

$$\delta^{a}{}_{b}\,\omega_{ca} = \omega_{cb} \tag{1.65}$$

```
> etaid[1] := eta[-f,-g,-a,-e] = eta[-a,-f,-g,-e] : T(%);
```

$$\eta_{fgae} = \eta_{afge} \tag{1.66}$$

```
> temp1 := TEDS(etaid[1], eq[10 b]) : T(%);
```

$$\omega^{a} u^{e}\,\eta_{afge} = \frac{1}{2}\,\eta^{a\ b\ c\ d}\, u_{b}\,\omega_{cd}\, u^{e}\,\eta_{afge} \tag{1.67}$$

```
> temp2 := eta[-a,-f,-g,-e]·eta[a, b, c, d] = -6·antisymm(delta[b,-f]·delta[c,-g]·delta[d,
    -e], b, d) : T(%);
```

$$\eta_{afge}\,\eta^{a\ b\ c\ d} = -\delta^{c}{}_{f}\,\delta^{d}{}_{g}\,\delta^{b}{}_{e} + \delta^{d}{}_{f}\,\delta^{c}{}_{g}\,\delta^{b}{}_{e} + \delta^{b}{}_{f}\,\delta^{d}{}_{g}\,\delta^{c}{}_{e} - \delta^{b}{}_{f}\,\delta^{c}{}_{g}\,\delta^{d}{}_{e} - \delta^{d}{}_{f}\,\delta^{b}{}_{g}\,\delta^{c}{}_{e} + \delta^{c}{}_{f}\,\delta^{b}{}_{g}\,\delta^{d}{}_{e} \tag{1.68}$$

> temp3 := expand(TEDS(temp2, temp1)) : T(%);

$$\omega^{a} u^{e} \eta_{afge} = -\frac{1}{2} \delta^{b}{}_{e} \delta^{c}{}_{f} \delta^{d}{}_{g} \omega_{cd} u^{e} u_{b} + \frac{1}{2} \delta^{b}{}_{e} \delta^{c}{}_{g} \delta^{d}{}_{f} \omega_{cd} u^{e} u_{b} + \frac{1}{2} \delta^{b}{}_{f} \delta^{c}{}_{e} \delta^{d}{}_{g} \omega_{cd} u^{e} u_{b} - \frac{1}{2} \delta^{b}{}_{f} \delta^{c}{}_{g} \delta^{d}{}_{e} \omega_{cd} u^{e} u_{b} - \frac{1}{2} \delta^{b}{}_{g} \delta^{c}{}_{e} \delta^{d}{}_{f} \omega_{cd} u^{e} u_{b} + \frac{1}{2} \delta^{b}{}_{g} \delta^{c}{}_{f} \delta^{d}{}_{e} \omega_{cd} u^{e} u_{b} \quad (1.69)$$

> temp4 := Absorbd(Absorbd(Absorbd(temp3))) : T(%);

$$\omega^{a} u^{e} \eta_{afge} = -\frac{1}{2} \omega_{fg} u^{e} u_{e} + \frac{1}{2} \omega_{gf} u^{e} u_{e} + \frac{1}{2} \omega_{eg} u^{e} u_{f} - \frac{1}{2} \omega_{ge} u^{e} u_{f} - \frac{1}{2} \omega_{ef} u^{e} u_{g} + \frac{1}{2} \omega_{fe} u^{e} u_{g} \quad (1.70)$$

> temp5 := expand(TEDS(u[e]·omega[-e,-f] = 0, temp4)) : T(%);

$$\omega^{a} u^{e} \eta_{afge} = -\frac{1}{2} \omega_{fg} u^{e} u_{e} + \frac{1}{2} \omega_{gf} u^{e} u_{e} + \frac{1}{2} \omega_{eg} u^{e} u_{f} - \frac{1}{2} \omega_{ge} u^{e} u_{f} + \frac{1}{2} \omega_{fe} u^{e} u_{g} \quad (1.71)$$

> temp6 := expand(TEDS(u[e]·omega[-g,-e] = 0, temp5)) : T(%);

$$\omega^{a} u^{e} \eta_{afge} = -\frac{1}{2} \omega_{fg} u^{e} u_{e} + \frac{1}{2} \omega_{gf} u^{e} u_{e} + \frac{1}{2} \omega_{eg} u^{e} u_{f} + \frac{1}{2} \omega_{fe} u^{e} u_{g} \quad (1.72)$$

> temp7 := expand(TEDS(u[e]·omega[-e,-g] = 0, temp6)) : T(%);

$$\omega^{a} u^{e} \eta_{afge} = -\frac{1}{2} \omega_{fg} u^{e} u_{e} + \frac{1}{2} \omega_{gf} u^{e} u_{e} + \frac{1}{2} \omega_{fe} u^{e} u_{g} \quad (1.73)$$

> temp8 := expand(TEDS(u[e]·omega[-f,-e] = 0, temp7)) : T(%);

$$\omega^{a} u^{e} \eta_{afge} = -\frac{1}{2} \omega_{fg} u^{e} u_{e} + \frac{1}{2} \omega_{gf} u^{e} u_{e} \quad (1.74)$$

> temp9 := expand(TEDS(u[e]·u[-e] =-1, temp8)) : T(%);

$$\omega^{a} u^{e} \eta_{afge} = \frac{1}{2} \omega_{fg} - \frac{1}{2} \omega_{gf} \quad (1.75)$$

> temp10 := expand(TEDS(omega[-g,-f] =-omega[-f,-g], temp9)) : T(%);

$$\omega^{a} u^{e} \eta_{afge} = \omega_{fg} \quad (1.76)$$

> temp11 := expand(TEDS(eta[-a,-f,-g,-e] = eta[-f,-g,-a,-e], temp10)) : T(%);

$$\eta_{fgae} \omega^{a} u^{e} = \omega_{fg} \quad (1.77)$$

> temp12 := subs(a =-a, e =-e, f =-f, g =-g, temp11) : T(%);

$$\eta^{fgae} \omega_{a} u_{e} = \omega^{fg} \quad (1.78)$$

completing the proof

>

>

****

****

End of page 1b - equations 7-10

****

>

```
> save deq, "deqs1b";
> save deltaid, "deltaid";
> save eq, "Seneqs1b";
> read "Seneqs1b" :
> show(eq);
```

$$table\left(\left[1 = \left(TensorPack{:-}T_{-a,\,-b} = \rho\, u_{-a}\, u_{-b}\right),\, 2 = \left(P_{-a,\,-b} = u_{-a}\, u_{-b} + g_{-a,\,-b}\right),\, 3 = \left(P_{a,\,-b}\, u_{b} = 0\right),\, 4 = \left(dX_{a} = u_{b}\, X_{a,\,-B}\right),\, 5 = \left(du_{a} = u_{b}\, u_{a,\,-B}\right),\, 6 = \left(u_{-a,\,-B} = \frac{1}{3}\,\theta\, P_{-a,\,-b} + \sigma_{-a,\,-b} + \omega_{-a,\,-b} - du_{-a}\, u_{-b}\right),\, 7 = \left(\theta = u_{a,\,-A}\right),\, 9 = \left(\omega_{-a,\,-b} = \frac{1}{2}\, P_{-a,\,c}\, P_{-b,\,d}\, u_{-c,\,-D} - \frac{1}{2}\, P_{-b,\,c}\, P_{-a,\,d}\, u_{-c,\,-D}\right),\, 8 = \left(\sigma_{-a,\,-b} = \frac{1}{2}\, P_{-a,\,c}\, P_{-b,\,d}\, u_{-c,\,-D} + \frac{1}{2}\, P_{-b,\,c}\, P_{-a,\,d}\, u_{-c,\,-D} - \frac{1}{3}\,\theta\, P_{-a,\,-b}\right),\, 10 = \left(\omega_{a} = \frac{1}{2}\,\eta_{a,\,b,\,c,\,d}\, u_{-b}\, \omega_{-c,\,-d}\right),\, 10\,a = \left(\omega_{a}\, u_{e} = \frac{1}{2}\,\eta_{a,\,b,\,c,\,d}\, u_{-b}\, \omega_{-c,\,-d}\, u_{e}\right),\, 10\,b = \left(\eta_{-f,\,-g,\,-a,\,-e}\, \omega_{a}\, u_{e} = \frac{1}{2}\,\eta_{-f,\,-g,\,-a,\,-e}\, \eta_{a,\,b,\,c,\,d}\, u_{-b}\, \omega_{-c,\,-d}\, u_{e}\right),\, 7\,a = \left(\theta = u_{d,\,-D}\right)\right]\right) \qquad \textbf{(1.79)}$$

>

```
> PrintSubArray(eq, 1, 10, y);
```

$$1,\ T_{a\,b} = \rho\, u_{a}\, u_{b}$$

$$2,\ P_{a\,b} = u\, u_{b\,a} + g_{a\,b}$$

$$3,\ P^{a}{}_{b}\, u^{b} = 0$$

$$4,\ dX^{a} = u^{b}\, X^{a}{}_{;b}$$

$$5,\ du^{a} = u^{b}\, u^{a}{}_{;b}$$

$$6,\ u_{a\,;b} = \frac{1}{3}\,\theta\, P_{a\,b} + \sigma_{a\,b} + \omega_{a\,b} - du_{a}\, u_{b}$$

$$7,\ \theta = u^{a}{}_{;a}$$

$$8, \sigma_{ab} = \frac{1}{2} P_a{}^c P_b{}^d u_{c;d} + \frac{1}{2} P_b{}^c P_a{}^d u_{c;d} - \frac{1}{3} \theta P_{ab}$$

$$9, \omega_{ab} = \frac{1}{2} P_a{}^c P_b{}^d u_{c;d} - \frac{1}{2} P_b{}^c P_a{}^d u_{c;d}$$

$$10, \omega^a = \frac{1}{2} \eta^{abcd} u_b \omega_{cd} \tag{1.80}$$

>