```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index) :
```

Chapter XX - Tensor analysis using indices - Senovilla et al. - Shearfree for dust page 3

if $\sigma_{a\,b}$=0 =>ωΘ=0

Author: Peter Huf

file 5 - eqs 44-48 - complete

```
> read "EFE" : read "SFE" :read "fids" :read "eqs2" :read "Seneqs4" :
```

```
> eq[42] := expand((mu + (16/3)·omega·omega)P[-a, b]·theta[-B] = (1/2)·omega[-a, c]·omega[-c, b]·theta[-B] + (theta/3)·P[-a, b]·mu[-B]) : T(%);
```

$$P_a{}^b\,\theta_{;b}\,\mu + \frac{16}{3}\,P_a{}^b\,\theta_{;b}\,\omega^2 = \frac{1}{2}\,\omega_a{}^c\,\omega_c{}^b\,\theta_{;b} + \frac{1}{3}\,\theta\,P_a{}^b\,\mu_{;b} \quad (1.1)$$

```
>
>
```

proof complete

****

```
>
>
>
```

****

Equation 44 leading to Equations 45 and 46

****

The original eq44 is

```
> eq[44] := (1/2)·theta·(mu + p - (16/3)·ω^2)·omega[-a, b]·theta[-B] - (1/4)·omega[-a, d]·omega[-d, c]·omega[-c, b]·theta[-B] + ((1/3)·theta·theta - omega·omega + (1/4)·(mu + 3·p))omega[-a, c]·omega[-c, b]·theta[-B] + ((32/9)·theta·theta·omega·omega + (mu + p - (16/3)·omega·omega)·((mu + 3·p)/2 - 2·omega·omega))·P[-a, b]·theta[-B] = 0 : T(%);
```

$$\frac{1}{2}\,\theta\left(\mu + p - \frac{16}{3}\,\omega^2\right)\omega_a{}^b\,\theta_{;b} - \frac{1}{4}\,\omega_a{}^d\,\omega_d{}^c\,\omega_c{}^b\,\theta_{;b} + \left(\frac{1}{3}\,\theta^2 - \omega^2 + \frac{1}{4}\,\mu \quad (1.2)$$

$$+\frac{3}{4}p\Big)\omega_a{}^c\omega_c{}^b\theta_{;b}+\left(\frac{32}{9}\theta^2\omega^2+\left(\mu+p-\frac{16}{3}\omega^2\right)\left(\frac{1}{2}\mu+\frac{3}{2}p-2\omega^2\right)\right)P_a{}^b\theta_{;b}=0$$

>

We start with eqs 42 and 43:

```
> eq[42] := expand((mu - (16/3)·omega·omega)P[-a, b]·theta[-B] = (1/2)·omega[-a, c]
   ·omega[-c, b]·theta[-B] + (theta/3)·P[-a, b]·mu[-B]) : T(%);
```

$$P_a{}^b\theta_{;b}\mu-\frac{16}{3}P_a{}^b\theta_{;b}\omega^2=\frac{1}{2}\omega_a{}^c\omega_c{}^b\theta_{;b}+\frac{1}{3}\theta P_a{}^b\mu_{;b} \tag{1.3}$$

>

```
> eq[43] := (1/2)·(mu + p - (16/3)·ω^2)·omega[-a, b]·theta[-B] + theta·(((112/9)
   ·omega·omega - (5/3)·(mu + p))·P[-a, b]·theta[-B] + (((5/9)·theta·theta - (2/3)
   ·omega·omega + (1/6)·(mu + 3·p))P[-a, b]·mu[-B] + (7/6)·theta·omega[-a, c]
   ·omega[-c, b]·theta[-B] - (1/4 + k)·omega[-a, d]·omega[-d, c]·omega[-c, b]
   ·theta[-B] = 0 : T(%);
```

$$\frac{1}{2}\left(\mu+p-\frac{16}{3}\omega^2\right)\omega_a{}^b\theta_{;b}+\theta\left(\frac{112}{9}\omega^2-\frac{5}{3}\mu-\frac{5}{3}p\right)P_a{}^b\theta_{;b}+\left(\frac{5}{9}\theta^2-\frac{2}{3}\omega^2+\frac{1}{6}\mu+\frac{1}{2}p\right)P_a{}^b\mu_{;b}+\frac{7}{6}\theta\omega_a{}^c\omega_c{}^b\theta_{;b}-\left(\frac{1}{4}+k\right)\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}=0 \tag{1.4}$$

```
> eq[43] : T(%);
```

$$\frac{1}{2}\left(\mu+p-\frac{16}{3}\omega^2\right)\omega_a{}^b\theta_{;b}+\theta\left(\frac{112}{9}\omega^2-\frac{5}{3}\mu-\frac{5}{3}p\right)P_a{}^b\theta_{;b}+\left(\frac{5}{9}\theta^2-\frac{2}{3}\omega^2+\frac{1}{6}\mu+\frac{1}{2}p\right)P_a{}^b\mu_{;b}+\frac{7}{6}\theta\omega_a{}^c\omega_c{}^b\theta_{;b}-\left(\frac{1}{4}+k\right)\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}=0 \tag{1.5}$$

```
> temp := expand(theta·eq[43]) : T(%);
```

$$\frac{1}{2}\theta\omega_a{}^b\theta_{;b}\mu+\frac{1}{2}\theta\omega_a{}^b\theta_{;b}p-\frac{8}{3}\theta\omega^2\omega_a{}^b\theta_{;b}+\frac{112}{9}\theta^2P_a{}^b\theta_{;b}\omega^2-\frac{5}{3}\theta^2P_a{}^b\theta_{;b}\mu-\frac{5}{3}\theta^2P_a{}^b\theta_{;b}p+\frac{5}{9}P_a{}^b\mu_{;b}\theta^3-\frac{2}{3}\theta P_a{}^b\mu_{;b}\omega^2 \tag{1.6}$$

$$+\frac{1}{6}\,\theta P_a{}^b \mu_{;b}\mu+\frac{1}{2}\,\theta P_a{}^b\mu_{;b}p+\frac{7}{6}\,\theta^2\omega_a{}^c\omega_c{}^b\theta_{;b}$$
$$-\frac{1}{4}\,\theta\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}-\theta\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}k=0$$

> *temp3* := *isolate*(*eq*[42], theta·*P*[-*a*, *b*]·mu[-*B*]) : *T*(%);

$$\theta P_a{}^b\mu_{;b}=3\,P_a{}^b\theta_{;b}\mu-16\,P_a{}^b\theta_{;b}\omega^2-\frac{3}{2}\,\omega_a{}^c\omega_c{}^b\theta_{;b} \qquad \textbf{(1.7)}$$

> *temp4* := *subs*(*p* = 0, *expand*(*TEDS*(*temp3*, *temp*))) : *T*(%);

$$\frac{1}{3}\,\theta^2\omega_a{}^c\omega_c{}^b\theta_{;b}+\frac{32}{9}\,\theta^2P_a{}^b\theta_{;b}\omega^2+\frac{1}{2}\,\theta\omega_a{}^b\theta_{;b}\mu-\frac{8}{3}\,\theta\omega^2\omega_a{}^b\theta_{;b} \qquad \textbf{(1.8)}$$
$$-\frac{1}{4}\,\theta\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}-\theta\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}k+\frac{32}{3}\,P_a{}^b\omega^4\theta_{;b}$$
$$-\frac{14}{3}\,P_a{}^b\mu\omega^2\theta_{;b}+\frac{1}{2}\,P_a{}^b\mu^2\theta_{;b}+\omega^2\omega_a{}^c\omega_c{}^b\theta_{;b}-\frac{1}{4}\,\mu\omega_a{}^c\omega_c{}^b\theta_{;b}=0$$

> *eq*[44] := *subs*(*p* = 0, *collect*(*expand*(*temp4*), [omega[-*a*, *b*], theta[-*B*], omega[-*a*, *d*], omega[-*d*, *c*], omega[-*c*, *b*], omega[-*a*, *c*], *P*[-*a*, *b*]], `*distributed*`)) : *T*(%);

$$\left(\frac{1}{3}\,\theta^2+\omega^2-\frac{1}{4}\,\mu\right)\omega_a{}^c\omega_c{}^b\theta_{;b}+\left(-\frac{1}{4}\,\theta-\theta k\right)\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}+\left(\frac{32}{9}\,\theta^2\omega^2 \qquad \textbf{(1.9)}$$
$$+\frac{32}{3}\,\omega^4-\frac{14}{3}\,\mu\omega^2+\frac{1}{2}\,\mu^2\right)P_a{}^b\theta_{;b}+\left(\frac{1}{2}\,\theta\mu-\frac{8}{3}\,\theta\omega^2\right)\omega_a{}^b\theta_{;b}=0$$

> *proof*[*eq44*] := *eq*[44] : *T*(%);

$$\left(\frac{1}{3}\,\theta^2+\omega^2-\frac{1}{4}\,\mu\right)\omega_a{}^c\omega_c{}^b\theta_{;b}+\left(-\frac{1}{4}\,\theta-\theta k\right)\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}+\left(\frac{32}{9}\,\theta^2\omega^2 \qquad \textbf{(1.10)}$$
$$+\frac{32}{3}\,\omega^4-\frac{14}{3}\,\mu\omega^2+\frac{1}{2}\,\mu^2\right)P_a{}^b\theta_{;b}+\left(\frac{1}{2}\,\theta\mu-\frac{8}{3}\,\theta\omega^2\right)\omega_a{}^b\theta_{;b}=0$$

>

contracting with w^a

> *proof*[*eq44c*] := *expand*(omega[*a*]·*proof*[*eq44*]) : *T*(%);

$$\frac{1}{3}\,\omega^a\theta^2\omega_a{}^c\omega_c{}^b\theta_{;b}+\frac{32}{9}\,\omega^a\theta^2P_a{}^b\theta_{;b}\omega^2+\frac{1}{2}\,\omega^a\theta\omega_a{}^b\theta_{;b}\mu \qquad \textbf{(1.11)}$$
$$-\frac{8}{3}\,\omega^a\theta\omega^2\omega_a{}^b\theta_{;b}-\frac{1}{4}\,\omega^a\theta\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}$$
$$-\omega^a\theta\omega_a{}^d\omega_d{}^c\omega_c{}^b\theta_{;b}k+\frac{32}{3}\,\omega^aP_a{}^b\omega^4\theta_{;b}-\frac{14}{3}\,\omega^aP_a{}^b\mu\omega^2\theta_{;b}$$
$$+\frac{1}{2}\,\omega^aP_a{}^b\mu^2\theta_{;b}+\omega^a\omega^2\omega_a{}^c\omega_c{}^b\theta_{;b}-\frac{1}{4}\,\omega^a\mu\omega_a{}^c\omega_c{}^b\theta_{;b}=0$$

> *eq*[42] : *T*(%);

$$P_a{}^b\theta_{;b}\mu-\frac{16}{3}\,P_a{}^b\theta_{;b}\omega^2=\frac{1}{2}\,\omega_a{}^c\omega_c{}^b\theta_{;b}+\frac{1}{3}\,\theta P_a{}^b\mu_{;b} \qquad \textbf{(1.12)}$$

> temp := isolate(eq[42], theta·P[-a, b]·mu[-B]) : T(%);

$$\theta P_a{}^b \mu_{;b} = 3 P_a{}^b \theta_{;b} \mu - 16 P_a{}^b \theta_{;b} \omega^2 - \frac{3}{2} \omega_a{}^c \omega_c{}^b \theta_{;b} \quad (1.13)$$

> proof[eq44b] := expand(TEDS(temp, proof[eq44c])) : T(%);

$$\frac{1}{3} \omega^a \theta^2 \omega_a{}^c \omega_c{}^b \theta_{;b} + \frac{32}{9} \omega^a \theta^2 P_a{}^b \theta_{;b} \omega^2 + \frac{1}{2} \omega^a \theta \omega_a{}^b \theta_{;b} \mu - \frac{8}{3} \omega^a \theta \omega^2 \omega_a{}^b \theta_{;b} - \frac{1}{4} \omega^a \theta \omega_a{}^d \omega_d{}^c \omega_c{}^b \theta_{;b} - \omega^a \theta \omega_a{}^d \omega_d{}^c \omega_c{}^b \theta_{;b} k + \frac{32}{3} \omega^a P_a{}^b \omega^4 \theta_{;b} - \frac{14}{3} \omega^a P_a{}^b \mu \omega^2 \theta_{;b} + \frac{1}{2} \omega^a P_a{}^b \mu^2 \theta_{;b} + \omega^a \omega^2 \omega_a{}^c \omega_c{}^b \theta_{;b} - \frac{1}{4} \omega^a \mu \omega_a{}^c \omega_c{}^b \theta_{;b} = 0 \quad (1.14)$$

>

>

> proof[eq44c] := expand(TEDS(omega[a]·omega[-a, b] = 0, proof[eq44b])) : T(%);

$$\frac{1}{3} \omega^a \theta^2 \omega_a{}^c \omega_c{}^b \theta_{;b} + \frac{32}{9} \omega^a \theta^2 P_a{}^b \theta_{;b} \omega^2 - \frac{1}{4} \omega^a \theta \omega_a{}^d \omega_d{}^c \omega_c{}^b \theta_{;b} - \omega^a \theta \omega_a{}^d \omega_d{}^c \omega_c{}^b \theta_{;b} k + \frac{32}{3} \omega^a P_a{}^b \omega^4 \theta_{;b} - \frac{14}{3} \omega^a P_a{}^b \mu \omega^2 \theta_{;b} + \frac{1}{2} \omega^a P_a{}^b \mu^2 \theta_{;b} + \omega^a \omega^2 \omega_a{}^c \omega_c{}^b \theta_{;b} - \frac{1}{4} \omega^a \mu \omega_a{}^c \omega_c{}^b \theta_{;b} = 0 \quad (1.15)$$

> proof[eq44d] := expand(TEDS(omega[a]·omega[-a, c] = 0, proof[eq44c])) : T(%);

$$\frac{32}{9} \omega^a \theta^2 P_a{}^b \theta_{;b} \omega^2 - \frac{1}{4} \omega^a \theta \omega_a{}^d \omega_d{}^c \omega_c{}^b \theta_{;b} - \omega^a \theta \omega_a{}^d \omega_d{}^c \omega_c{}^b \theta_{;b} k + \frac{32}{3} \omega^a P_a{}^b \omega^4 \theta_{;b} - \frac{14}{3} \omega^a P_a{}^b \mu \omega^2 \theta_{;b} + \frac{1}{2} \omega^a P_a{}^b \mu^2 \theta_{;b} = 0 \quad (1.16)$$

> proof[eq44e] := expand(TEDS(omega[a]·omega[-a, d] = 0, proof[eq44d])) : T(%);

$$\frac{32}{9} \omega^a \theta^2 P_a{}^b \theta_{;b} \omega^2 + \frac{32}{3} \omega^a P_a{}^b \omega^4 \theta_{;b} - \frac{14}{3} \omega^a P_a{}^b \mu \omega^2 \theta_{;b} + \frac{1}{2} \omega^a P_a{}^b \mu^2 \theta_{;b} = 0 \quad (1.17)$$

> proof[eq44f] := expand(TEDS(omega[a]·P[-a, b] = omega[b], proof[eq44e])) : T(%);

$$\frac{32}{9} \theta^2 \theta_{;b} \omega^2 \omega^b + \frac{32}{3} \omega^4 \theta_{;b} \omega^b - \frac{14}{3} \mu \omega^2 \theta_{;b} \omega^b + \frac{1}{2} \mu^2 \theta_{;b} \omega^b = 0 \quad (1.18)$$

> proof[eq44g] := collect(proof[eq44f], [omega[b], theta[-B]], `distributed`) : T(%);

$$\left(\frac{32}{9} \theta^2 \omega^2 + \frac{32}{3} \omega^4 - \frac{14}{3} \mu \omega^2 + \frac{1}{2} \mu^2\right) \omega^b \theta_{;b} = 0 \quad (1.19)$$

> eq[45] := proof[eq44g] : T(%);

$$\left(\frac{32}{9}\theta^2\omega^2+\frac{32}{3}\omega^4-\frac{14}{3}\mu\omega^2+\frac{1}{2}\mu^2\right)\omega^{b}\theta_{;b}=0 \qquad \textbf{(1.20)}$$

proof of eq45

From eq45, if $\omega^{b}\theta_{;b}\neq 0$ **then**

> eq[46] := collect((op(1, lhs(eq[45]))) = 0, [omega, mu], `distributed`) : T(%);

$$\frac{32}{9}\theta^2\omega^2+\frac{32}{3}\omega^4-\frac{14}{3}\mu\omega^2+\frac{1}{2}\mu^2=0 \qquad \textbf{(1.21)}$$

which is eq46

>

end of proof eqs 44 and 45 and 46

****
****

Equations 47 and 48

****

>

Two different possibilities appear from eq45:
case 1:

> #eq[46] := ((32/9)·theta·theta·omega·omega + (mu + p − (16/3)·omega·omega) ·((mu + 3·p)/2 − 2·omega·omega)) = 0 : T(%);

leads to eq47:

> eq[47] := ((64/9)·omega·omega·omega·omega − 2·mu·omega·omega − (38/3)·p·omega ·omega + p·(mu + p)) = 0 : T(%);

$$\frac{64}{9}\omega^4-2\mu\omega^2-\frac{38}{3}p\,\omega^2+p(\mu+p)=0 \qquad \textbf{(1.22)}$$

Time propogation of eq46 leads to:

> temp := dotT(eq[46]) : T(%);

$$\frac{64}{9}\theta\,dottheta\,\omega^2+\frac{64}{9}\theta^2\omega\,dotomega+\frac{128}{3}\omega^3\,dotomega-\frac{14}{3}\,dotmu\,\omega^2-\frac{28}{3}\mu\omega\,dotomega+\mu\,dotmu=0 \qquad \textbf{(1.23)}$$

> temp2 := TEDS(dotmu =−mu·theta, temp) : T(%);

$$\frac{64}{9}\theta\,dottheta\,\omega^2+\frac{64}{9}\theta^2\omega\,dotomega+\frac{128}{3}\omega^3\,dotomega+\frac{14}{3}\omega^2\theta\mu-\frac{28}{3}\mu\omega\,dotomega-\theta\mu^2=0 \qquad \textbf{(1.24)}$$

> *temp3* := *TEDS*(*dotomega* = $-\frac{2}{3}$ ·theta·omega, *temp2*) : *T*(%);

$$\frac{64}{9}\,\theta\, dottheta\, \omega^2 - \frac{128}{27}\,\theta^3\,\omega^2 - \frac{256}{9}\,\omega^4\,\theta + \frac{98}{9}\,\omega^2\,\theta\,\mu - \theta\,\mu^2 = 0 \qquad \textbf{(1.25)}$$

> *temp4* := *factor*(*TEDS*(*dottheta* = $-\frac{1}{3}$ · theta·theta + 2·omega·omega − $\frac{1}{2}$ ·mu, *temp3*)) : *T*(%);

$$-\frac{1}{9}\,\theta\left(128\,\omega^4 + 64\,\omega^2\,\theta^2 - 66\,\mu\,\omega^2 + 9\,\mu^2\right) = 0 \qquad \textbf{(1.26)}$$

Using eq46:

> *temp5* := *isolate*(*eq*[46], $\frac{32}{3}\cdot\omega^4$) : *T*(%);

$$\frac{32}{3}\,\omega^4 = -\frac{32}{9}\,\theta^2\,\omega^2 + \frac{14}{3}\,\mu\,\omega^2 - \frac{1}{2}\,\mu^2 \qquad \textbf{(1.27)}$$

> *temp6* := −2·(*temp5*) : *T*(%);

$$-\frac{64}{3}\,\omega^4 = \frac{64}{9}\,\theta^2\,\omega^2 - \frac{28}{3}\,\mu\,\omega^2 + \mu^2 \qquad \textbf{(1.28)}$$

> *eq*[47] := *temp6* :

> *temp7* := *factor*(*TEDS*(*temp6*, *temp4*)) : *T*(%);

$$-\frac{1}{27}\,\theta\left(64\,\omega^2\,\theta^2 - 30\,\mu\,\omega^2 + 9\,\mu^2\right) = 0 \qquad \textbf{(1.29)}$$

> *eq*[48] := *temp7* : *T*(%);

$$-\frac{1}{27}\,\theta\left(64\,\omega^2\,\theta^2 - 30\,\mu\,\omega^2 + 9\,\mu^2\right) = 0 \qquad \textbf{(1.30)}$$

>

either $\theta = 0$ **or** $\omega = 0$ **or** *the bracket is zero, which* **by** *lemma* 3 *proves* $\theta\,\omega = 0$

>

>

proof eq48 complete

> **save** *eq*, "Seneqs5";

> *PrintSubArray*(*eq*, 1, 48, *y*);

$$1,\, T_{a\,b} = \rho\, u_a\, u_b$$

$$2,\, P_{a\,b} = g_{a\,b} + u_a\, u_b$$

$$3,\, P^{`a}{}_{b}\, u^{`b} = 0$$

$$4,\, dX^{`a} = u^{`b}\, X^{`a}{}_{;b}$$

$$5,\ du^{a} = u^{b} u^{a}{}_{;b}$$

$$6,\ u_{a;b} = \frac{1}{3}\,\theta P_{ab} + \sigma_{ab} + \omega_{ab} - du_{a} u_{b}$$

$$7,\ \theta = u^{a}{}_{;a}$$

$$8,\ \sigma_{ab} = \frac{1}{2}\,P_{a}{}^{c} P_{b}{}^{d} u_{c;d} + \frac{1}{2}\,P_{b}{}^{c} P_{a}{}^{d} u_{c;d} - \frac{1}{3}\,\theta P_{ab}$$

$$9,\ \omega_{ab} = \frac{1}{2}\,P_{a}{}^{c} P_{b}{}^{d} u_{c;d} - \frac{1}{2}\,P_{b}{}^{c} P_{a}{}^{d} u_{c;d}$$

$$10,\ \omega^{a} = \frac{1}{2}\,\eta^{abcd} u_{b} \omega_{cd}$$

$$11,\ \omega_{ab} = \eta_{abef} \omega^{e} u^{f}$$

$$12,\ \omega^{2} = \frac{1}{2}\,\omega^{ab} \omega_{ab}$$

"string"

13, "iff(iff(omega[-a,-b] = 0,omega[-a]),omega = 0)"

$$14,\ \omega_{a}{}^{c} \omega_{c}{}^{b} = \omega_{a} \omega^{b} - \omega^{2} P_{a}{}^{b}$$

$$15,\ \frac{1}{2}\,u_{b;a} - \frac{1}{2}\,u_{a;b} = \frac{1}{2}\,du_{a} u_{b} - \frac{1}{2}\,du_{b} u_{a} + \omega^{ab}$$

$$16,\ \frac{1}{6}\,u_{c} u_{b;a} - \frac{1}{6}\,u_{c} u_{a;b} + \frac{1}{6}\,u_{b} u_{a;c} - \frac{1}{6}\,u_{b} u_{c;a} + \frac{1}{6}\,u_{a} u_{c;b} - \frac{1}{6}\,u_{a} u_{b;c} = 0$$

$$17,\ \sigma_{ab} = 0$$

$$18,\ u_{a;b} = \frac{1}{3}\,\theta P_{ab} + \omega_{ab}$$

$$19,\ u^{a}{}_{;c;d} - u^{a}{}_{;d;c} = R^{a}{}_{bcd} u^{b}$$

$$20,\ dottheta + \frac{1}{3}\,\theta^{2} - 2\,\omega^{2} + \frac{1}{2}\,\mu = 0$$

$$21,\ P_{a}{}^{c} P_{b}{}^{d} \omega_{cd;f} u^{f} + \frac{2}{3}\,\theta\,\omega_{ab} = 0$$

$$22,\ \omega_{a} \omega_{b} - \frac{1}{3}\,P_{ab} \omega^{2} + E_{ab} = 0$$

$$23,\ E_{ab} = C_{abcd} u^{c} u^{d}$$

$$24,\ H_{ab} = \frac{1}{2}\,\eta_{ae}{}^{cd} C_{cdbf} u^{e} u^{f}$$

$$25,\ P^{a}{}_{b} \omega^{b}{}_{;f} u^{f} + \frac{2}{3}\,\theta\,\omega^{a} = 0$$

$$26,\ 2\,P^{ab} \theta_{;b} + 3\,P^{a}{}_{b} \omega^{bd}{}_{;d} = 0$$

$$27,\ \omega^{a}{}_{;a}=0$$

$$28,\ H_{ab}=\frac{1}{2}\,P_{a}{}^{c}P_{b}{}^{d}\omega_{d}{}^{;c}+\frac{1}{2}\,P_{b}{}^{c}P_{a}{}^{d}\omega_{d}{}^{;c}$$

$$29,\ \omega_{ab}\,\omega^{b}{}^{c}{}_{;c}=P_{a}{}^{b}\,\omega^{c}\,\omega_{b;c}-P_{a}{}^{b}\,\omega^{c}\,\omega_{c;b}$$

$$30,\ dotmu+\theta\,\mu=0$$

$$31,\ (\mu+p)\,du^{a}+P^{a}{}^{b}\,p_{;b}$$

$$32,\ du^{a}=0$$

$$33,\ u_{a}=-\frac{f_{;a}}{fdot}$$

$$34,\ \mu=(c1-1)\,p+c2\,\omega^{2}$$

$$35,\ dotomega_{ab}=-\frac{2}{3}\,\theta\,\omega_{ab}$$

$$36,\ dotomega=-\frac{2}{3}\,\theta\,\omega$$

$$37,\ \theta\left(c1p-\frac{1}{3}\,c2\,\omega^{2}\right)=0$$

$$38,\ \frac{\partial}{\partial t}\left(P^{a}{}^{b}f_{;b}\right)=P^{a}{}^{b}f'_{;b}+\omega^{a}{}^{b}f_{;b}-\frac{1}{3}\,\theta\,P^{a}{}^{b}f_{;b}$$

$$39,\ 2\,P_{a}{}^{b}\mu_{;b}-13\,P_{a}{}^{b}\,\omega^{c}\,\omega_{c;b}-3\,P_{a}{}^{b}\,\omega^{c}\,\omega_{b;c}=0$$

$$40,\ P_{a}{}^{b}\mu_{;b}-8\,\omega\,P_{a}{}^{b}\,\omega_{;b}+\omega_{a}{}^{b}\,\theta_{;b}=0$$

$$41,\ -8\,\omega_{a}{}^{b}\,\omega\,\omega_{;b}+\omega_{a}{}^{b}\,\mu_{;b}+\omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}=0$$

$$42,\ P_{a}{}^{b}\,\theta_{;b}\,\mu-\frac{16}{3}\,P_{a}{}^{b}\,\theta_{;b}\,\omega^{2}=\frac{1}{2}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}+\frac{1}{3}\,\theta\,P_{a}{}^{b}\,\mu_{;b}$$

$$43,\ \frac{1}{2}\left(\mu+p-\frac{16}{3}\,\omega^{2}\right)\omega_{a}{}^{b}\,\theta_{;b}+\theta\left(\frac{112}{9}\,\omega^{2}-\frac{5}{3}\,\mu-\frac{5}{3}\,p\right)P_{a}{}^{b}\,\theta_{;b}+\left(\frac{5}{9}\,\theta^{2}-\frac{2}{3}\,\omega^{2}+\frac{1}{6}\,\mu+\frac{1}{2}\,p\right)P_{a}{}^{b}\,\mu_{;b}+\frac{7}{6}\,\theta\,\omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}-\left(\frac{1}{4}+k\right)\omega_{a}{}^{d}\,\omega_{d}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}=0$$

$$44,\ \left(-\frac{1}{4}\,\theta-\theta\,k\right)\omega_{a}{}^{d}\,\omega_{d}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}+\left(\frac{1}{3}\,\theta^{2}-\frac{1}{4}\,\mu+\omega^{2}\right)\omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}+\left(\frac{1}{2}\,\theta\,\mu-\frac{8}{3}\,\theta\,\omega^{2}\right)\omega_{a}{}^{b}\,\theta_{;b}+\left(\frac{32}{3}\,\omega^{4}+\frac{1}{2}\,\mu^{2}+\frac{32}{9}\,\theta^{2}\,\omega^{2}-\frac{14}{3}\,\omega^{2}\,\mu\right)\theta_{;b}\,P_{a}{}^{b}=0$$

$$45, \left(\frac{32}{3}\,\omega^4+\frac{1}{2}\,\mu^2+\frac{32}{9}\,\theta^2\,\omega^2-\frac{14}{3}\,\omega^2\,\mu\right)\omega^{`b}\,\theta_{;b}=0$$

$$46, \frac{32}{3}\,\omega^4+\frac{1}{2}\,\mu^2+\frac{32}{9}\,\theta^2\,\omega^2-\frac{14}{3}\,\omega^2\,\mu=0$$

$$47, \frac{64}{3}\,\omega^4=-\mu^2-\frac{64}{9}\,\theta^2\,\omega^2+\frac{28}{3}\,\omega^2\,\mu$$

$$48, -\frac{1}{27}\,\theta\left(9\,\mu^2+64\,\theta^2\,\omega^2-30\,\omega^2\,\mu\right)=0 \qquad (1.31)$$

****

Equation 48 - double check here - probably not needed here

****

>

leads to eq48, by time propogation:

> $eq[48] := \left(\mathrm{mu}+\left(\frac{7}{3}\right)\cdot p-\left(\frac{40}{9}\right)\cdot \mathrm{omega}\cdot\mathrm{omega}\right)\cdot\mathrm{omega}\cdot\mathrm{omega}=0 : T(\%);$

$$\left(\mu+\frac{7}{3}\,p-\frac{40}{9}\,\omega^2\right)\omega^2=0 \qquad (1.32)$$

then either ω=0 or by lemma 3 ωθ=0

case 2:
if $\omega^{b}\,\theta_{;b}=0$ then eq44 leads to (with identities (14))

> $eq[49] := \left(\frac{\mathrm{theta}}{2}\right)\cdot\left(29\cdot\omega\cdot\mathrm{omega}-6\cdot(\mathrm{mu}+p)\right)\cdot\mathrm{omega}[-a,b]\cdot\mathrm{theta}[-B]=\left(\left(\frac{58}{3}\right)\cdot\mathrm{theta}\cdot\theta\cdot\omega\cdot\mathrm{omega}+\left(2\cdot\omega\cdot\mathrm{omega}-\frac{(\mathrm{mu}+3\,p)}{2}\right)\cdot\left(29\cdot\omega\cdot\mathrm{omega}-6\cdot(\mathrm{mu}+p)\right)\right)\cdot P[-a,b]\cdot\mathrm{theta}[-B] : T(\%);$

$$\frac{1}{2}\,\theta\left(29\,\omega^2-6\,\mu-6\,p\right)\omega_a{}^{`b}\,\theta_{;b}=\left(\frac{58}{3}\,\theta^2\,\omega^2+\left(2\,\omega^2-\frac{1}{2}\,\mu-\frac{3}{2}\,p\right)\left(29\,\omega^2-6\,\mu-6\,p\right)\right)P_a{}^{`b}\,\theta_{;b} \qquad (1.33)$$

the LHS of eq49 must vanish since, they are orthogonal:

> $eq[50\,a] := \mathrm{omega}[-a,b]\cdot\mathrm{theta}[-B]=\mathrm{omega}[-a,-c]\cdot P[c,b]\cdot\mathrm{theta}[-B] : T(\%);$

$$\omega_a{}^{`b}\,\theta_{;b}=\omega_{a\,c}\,P^{`c`b}\,\theta_{;b} \qquad (1.34)$$

and

> $eq[50\,b] := \mathrm{omega}[-a,b]\cdot\mathrm{theta}[-B]=\mathrm{eta}[-a,-c,-d,-e]\cdot\mathrm{omega}[d]\cdot u[c]\cdot P[c,b]\cdot\mathrm{theta}[-B] : T(\%);$

$$\omega_a{}^{`b}\,\theta_{;b}=\eta_{a\,c\,d\,e}\,\omega^{`d}\,u^{`c}\,P^{`c`b}\,\theta_{;b} \qquad (1.35)$$

by argument leads to $\omega\theta=0$

> **save** *eq*, "Seneqs5";

>

>

>

>