```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index);
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for dust
page 3

if $\sigma_{a\,b}$**=0 =>ωΘ=0**

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file 3 - eq42

In this file we continue to follow the equations outlined by Senovilla et al. (2007) with the assumptions for dust
i.e

```
> read "EFE" : read "SFE" :read "fids" :read "eqs2" :read "Seneqs3c" :
>
```

****

Equation 42

****

The original eq42 is

```
> eq[42] := (mu + (16/3)·omega·omega)·P[a, b]·theta[-B] = (1/2)·omega[-a, c]·omega[-c, b]·theta[-B] + (theta/3)·P[-a, b]·mu[-B] : T(%);
```

$$\left(\mu+\frac{16}{3}\,\omega^2\right)P^{a\;b}\theta_{;b}=\frac{1}{2}\,\omega_a{}^c\omega_c{}^b\theta_{;b}+\frac{1}{3}\,\theta P_a{}^b\mu_{;b} \tag{1.1}$$

Proof of eq42:
The first step is the time dilation of eq40

```
> eq[40] : T(%);
```

$$-8\,\omega P_a{}^b\omega_{;b}+P_a{}^b\mu_{;b}+\omega_a{}^b\theta_{;b}=0 \tag{1.2}$$

for the first term we use eq38

```
> eq[38] := Diff(P[a, b]·f[-B], t) = P[a, b]·f`'`[-B] + omega[a, b]·f[-B] − (1/3)· theta·P[a, b]·f[-B] : T(%);
```

$$\frac{\partial}{\partial t}\left(P^{a\;b}f_{;b}\right)=P^{a\;b}f^{`\prime`}{}_{;b}+\omega^{a\;b}f_{;b}-\frac{1}{3}\,\theta P^{a\;b}f_{;b} \tag{1.3}$$

```
> T(eq[38]);
```

$$\frac{\partial}{\partial t}\left(P^{a\ b} f_{;b}\right) = P^{a\ b} f^{`\prime`}_{;b} + \omega^{a\ b} f_{;b} - \frac{1}{3}\,\theta\, P^{a\ b} f_{;b} \tag{1.4}$$

```
> proof[eq41t1] := subs(f=mu, eq[38]) : T(%);
```

$$\frac{\partial}{\partial t}\left(P^{a\ b} \mu_{;b}\right) = P^{a\ b} \mu^{`\prime`}_{;b} + \omega^{a\ b} \mu_{;b} - \frac{1}{3}\,\theta\, P^{a\ b} \mu_{;b} \tag{1.5}$$

we use the following nomenclature

```
> temp := mu`'`=mudot : T(%);
```

$$\mu^{`\prime`} = mudot \tag{1.6}$$

```
> temp2 := mu`'`[-b"]=mudot[-b"] : T(%);
```

$$\mu^{`\prime`}{}_{b''} = mudot_{b''} \tag{1.7}$$

```
> temp3 := TEDS(temp2, temp) : T(%);
```

$$\mu^{`\prime`} = mudot \tag{1.8}$$

```
> temp4 := mu`'`[-B]=cod(rhs(temp3),-b) : T(%);
```

$$\mu^{`\prime`}{}_{;b} = mudot_{;b} \tag{1.9}$$

we can use the format mudot=μ `'` *etc...*

```
> temp5 := TEDS(temp4, proof[eq41t1]) : T(%);
```

$$\frac{\partial}{\partial t}\left(P^{a\ b} \mu_{;b}\right) = P^{a\ b}\, mudot_{;b} + \omega^{a\ b} \mu_{;b} - \frac{1}{3}\,\theta\, P^{a\ b} \mu_{;b} \tag{1.10}$$

```
> temp6 := subs(a=-a, temp5) : T(%);
```

$$\frac{\partial}{\partial t}\left(P_{a}{}^{b} \mu_{;b}\right) = P_{a}{}^{b}\, mudot_{;b} + \mu_{;b}\, \omega_{a}{}^{b} - \frac{1}{3}\,\theta\, P_{a}{}^{b} \mu_{;b} \tag{1.11}$$

```
> term1 := rhs(temp6) : T(%);
```

$$P_{a}{}^{b}\, mudot_{;b} + \mu_{;b}\, \omega_{a}{}^{b} - \frac{1}{3}\,\theta\, P_{a}{}^{b} \mu_{;b} \tag{1.12}$$

****

```
> term2 := op(2, op(1, eq[40])) : T(%);
```

$$P_{a}{}^{b} \mu_{;b} \tag{1.13}$$

```
> temp := dotT(term2) : T(%);
```

$$P_{a}{}^{b}\, dotmu_{;b} + dotP_{a}{}^{b} \mu_{;b} \tag{1.14}$$

```
> temp2 := TEDS(dotP[-a, b]=0, temp) : T(%);
```

$$P_{a}{}^{b}\, dotmu_{;b} \tag{1.15}$$

using eq36b

```
> temp3 := TEDS(dotomega=-2/3·omega·theta, temp2) : T(%);
```

$$P_{a}{}^{b}\, dotmu_{;b} \tag{1.16}$$

> term2a := op(1, temp3) : T(%);

$$P_a{}^{b} \qquad \textbf{(1.17)}$$

> temp4 := op(2, temp3) =-8·omega· subs(f=omega, subs(a =-a, rhs(eq[38]))) : T(%);

$$dotmu_{;b} = -8\,\omega\left(P_a{}^{b}\,\omega`'`_{;b} + \omega_a{}^{b}\,\omega_{;b} - \frac{1}{3}\,\theta\,P_a{}^{b}\,\omega_{;b}\right) \qquad \textbf{(1.18)}$$

>

> temp5 := expand(TEDS(omega`'`[-B]=omegadot[-B], temp4)) : T(%);

$$dotmu_{;b} = -8\,\omega\,P_a{}^{b}\,omegadot_{;b} - 8\,\omega\,\omega_{;b}\,\omega_a{}^{b} + \frac{8}{3}\,\omega\,\theta\,P_a{}^{b}\,\omega_{;b} \qquad \textbf{(1.19)}$$

> temp6 := expand(TEDS(temp5, temp3)) : T(%);

$$\frac{8}{3}\,\omega\,\theta\,P2_{-a,\,b}\,\omega_{;b} - 8\,\omega\,P2_{-a,\,b}\,omegadot_{;b} - 8\,\omega\,P_a{}^{b}\,\omega_{;b}\,\omega_a{}^{b} \qquad \textbf{(1.20)}$$

> term2 := temp6 : T(%);

$$\frac{8}{3}\,\omega\,\theta\,P2_{-a,\,b}\,\omega_{;b} - 8\,\omega\,P2_{-a,\,b}\,omegadot_{;b} - 8\,\omega\,P_a{}^{b}\,\omega_{;b}\,\omega_a{}^{b} \qquad \textbf{(1.21)}$$

****

> term3 := op(3, op(1, eq[40])) : T(%);

$$\omega_a{}^{b}\,\theta_{;b} \qquad \textbf{(1.22)}$$

> temp := dotT(term3) : T(%);

$$\omega_a{}^{b}\,dottheta_{;b} + dotomega_a{}^{b}\,\theta_{;b} \qquad \textbf{(1.23)}$$

> temp2 := dotomega[-a, b] =- $\frac{2}{3}$ ·ω[-a, b]·theta : T(%);

$$dotomega_a{}^{b} = -\frac{2}{3}\,\omega_a{}^{b}\,\theta \qquad \textbf{(1.24)}$$

> temp3 := TEDS(temp2, temp) : T(%);

$$\omega_a{}^{b}\,dottheta_{;b} - \frac{2}{3}\,\theta_{;b}\,\omega_a{}^{b}\,\theta \qquad \textbf{(1.25)}$$

we can use the format thetadot=θ`'` **for** *scalars* **and** *dotheta*[ ] **for** *vectors* **and** *tensors etc*...

>

which, due to torsion free is:

> temp4 := dottheta[-B] = theta[-E,-B]·u[e] : T(%);

$$dottheta_{;b} = \theta_{;e\,;b}\,u^{e} \qquad \textbf{(1.26)}$$

Now for (dottheta), due to torsion-free

> temp4a := theta[-E,-B] = theta[-B,-E] : T(%);

$$\theta_{;e\,;b} = \theta_{;b\,;e} \qquad \textbf{(1.27)}$$

> temp5 := thetadot[-B] = cod(theta[-E]·u[e],-b) : T(%);

$$thetadot_{;b} = \theta_{;e} u^{e}{}_{;b} + \theta_{;e;b} u^{e} \qquad (1.28)$$

```
> temp6 := TEDS(temp4a, temp5) : T(%);
```

$$thetadot_{;b} = \theta_{;e} u^{e}{}_{;b} + u^{e} \theta_{;b;e} \qquad (1.29)$$

```
> temp7 := TEDS(theta[-B,-E]·u[e]=dottheta[-B], temp6) : T(%);
```

$$thetadot_{;b} = \theta_{;e} u^{e}{}_{;b} + dottheta_{;b} \qquad (1.30)$$

```
> #eq[18] := u[-a,-B] = (1/3)·P[-a,-b]·theta + omega[-a,-b] : T(%);
> T(eq[18]);
```

$$u_{a;b} = \frac{1}{3}\theta P_{ab} + \omega_{ab} \qquad (1.31)$$

```
> temp8 := subs(a=-e, eq[18]) : T(%);
```

$$u^{e}{}_{;b} = \frac{1}{3}\theta P^{e}{}_{b} + \omega^{e}{}_{b} \qquad (1.32)$$

```
> temp9 := TEDS(temp8, temp7) : T(%);
```

$$thetadot_{;b} = \frac{1}{3}\theta_{;e}\theta P^{e}{}_{b} + \theta_{;e}\omega^{e}{}_{b} + dottheta_{;b} \qquad (1.33)$$

```
> temp10 := TEDS(P[-b, e]=g[-b, e]+u[-b]·u[e], temp9) : T(%);
```

$$thetadot_{;b} = \frac{1}{3}\theta_{;e}\theta P^{e}{}_{b} + \theta_{;e}\omega^{e}{}_{b} + dottheta_{;b} \qquad (1.34)$$

```
> temp11 := Absorbg(temp10) : T(%);
```

$$thetadot_{;b} = \frac{1}{3}\theta_{;e}\theta P^{e}{}_{b} + \theta_{;e}\omega^{e}{}_{b} + dottheta_{;b} \qquad (1.35)$$

multiplying by

```
> temp12 := expand(omega[-a, b]·temp11) : T(%);
```

$$\omega_{a}{}^{b}\, thetadot_{;b} = \frac{1}{3}\omega_{a}{}^{b}\theta_{;e}\theta P^{e}{}_{b} + \omega_{a}{}^{b}\theta_{;e}\omega^{e}{}_{b} + \omega_{a}{}^{b}\, dottheta_{;b} \qquad (1.36)$$

```
> temp13 := expand(TEDS(omega[-a, b]·u[-b]=0, temp12)) : T(%);
```

$$\omega_{a}{}^{b}\, thetadot_{;b} = \frac{1}{3}\omega_{a}{}^{b}\theta_{;e}\theta P^{e}{}_{b} + \omega_{a}{}^{b}\theta_{;e}\omega^{e}{}_{b} + \omega_{a}{}^{b}\, dottheta_{;b} \qquad (1.37)$$

```
> temp14 := expand(TEDS(omega[-a, b]·omega[-b, e]·theta[-E]=omega[-a, c]·omega[
    -c, b]·theta[-B], temp13)) : T(%);
```

$$\omega_{a}{}^{b}\, thetadot_{;b} = \frac{1}{3}\omega_{a}{}^{b}\theta_{;e}\theta P^{e}{}_{b} + \omega_{a}{}^{b}\theta_{;e}\omega^{e}{}_{b} + \omega_{a}{}^{b}\, dottheta_{;b} \qquad (1.38)$$

```
> temp15 := rhs(temp14) - lhs(temp14) = 0 : T(%);
```

$$\frac{1}{3}\omega_{a}{}^{b}\theta_{;e}\theta P^{e}{}_{b} + \omega_{a}{}^{b}\theta_{;e}\omega^{e}{}_{b} + \omega_{a}{}^{b}\, dottheta_{;b} - \omega_{a}{}^{b}\, thetadot_{;b} = 0 \qquad (1.39)$$

```
> temp16 := op(1, op(1, temp15)) =-op(2, op(1, temp15)) - -op(3, op(1, temp15)) - op(4,
```

op(1, temp15)) : T(%);

$$\frac{1}{3}\,\omega_{a}{}^{b}\,\theta_{;e}\,\theta\,P^{e}{}_{b} = -\omega_{a}{}^{b}\,\theta_{;e}\,\omega^{e}{}_{b} + \omega_{a}{}^{b}\,dottheta_{;b} + \omega_{a}{}^{b}\,thetadot_{;b} \quad (1.40)$$

hence combining terms:

> term3 := expand(TEDS(temp16 , temp3)) : T(%);

$$\omega_{a}{}^{b}\,dottheta_{;b} - \frac{2}{3}\,\theta_{;b}\,\omega_{a}{}^{b}\,\theta \quad (1.41)$$

>

****

so we have the 3 terms from the time differentiation:

> T(term1);

$$P_{a}{}^{b}\,mudot_{;b} + \mu_{;b}\,\omega_{a}{}^{b} - \frac{1}{3}\,\theta\,P_{a}{}^{b}\,\mu_{;b} \quad (1.42)$$

> T(term2);

$$\frac{8}{3}\,\omega\,\theta\,P^{2}_{-a,\,b}\,\omega_{;b} - 8\,\omega\,P^{2}_{-a,\,b}\,omegadot_{;b} - 8\,\omega\,P_{a}{}^{b}\,\omega_{;b}\,\omega_{a}{}^{b} \quad (1.43)$$

> T(term3);

$$\omega_{a}{}^{b}\,dottheta_{;b} - \frac{2}{3}\,\theta_{;b}\,\omega_{a}{}^{b}\,\theta \quad (1.44)$$

> total := term1 + term2 + term3 = 0 : T(%);

$$P_{a}{}^{b}\,mudot_{;b} + \mu_{;b}\,\omega_{a}{}^{b} - \frac{1}{3}\,\theta\,P_{a}{}^{b}\,\mu_{;b} + \frac{8}{3}\,\omega\,\theta\,P^{2}_{-a,\,b}\,\omega_{;b} - 8\,\omega\,P^{2}_{-a,\,b}\,omegadot_{;b} - 8\,\omega\,P_{a}{}^{b}\,\omega_{;b}\,\omega_{a}{}^{b} + \omega_{a}{}^{b}\,dottheta_{;b} - \frac{2}{3}\,\theta_{;b}\,\omega_{a}{}^{b}\,\theta = 0 \quad (1.45)$$

>

>

>

>

Now using eq41:

> eq[41] := -8 * omega[-a, b] * omega * omega[-B] + omega[-a, b] * mu[-B] + omega[-a, c] * omega[-c, b] * theta[-B] = 0 : T(%);

$$-8\,\omega_{a}{}^{b}\,\omega\,\omega_{;b} + \omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b} + \mu_{;b}\,\omega_{a}{}^{b} = 0 \quad (1.46)$$

> temp := eq[41] : T(%);

$$-8\,\omega_{a}{}^{b}\,\omega\,\omega_{;b} + \omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b} + \mu_{;b}\,\omega_{a}{}^{b} = 0 \quad (1.47)$$

> total2 := lhs(total) - lhs(temp) = 0 : T(%);

$$P_{a}{}^{b}\,mudot_{;b} - \frac{1}{3}\,\theta\,P_{a}{}^{b}\,\mu_{;b} + \frac{8}{3}\,\omega\,\theta\,P^{2}_{-a,\,b}\,\omega_{;b} - 8\,\omega\,P^{2}_{-a,\,b}\,omegadot_{;b} \quad (1.48)$$

$$-8\,\omega\,P_a{}^b\,\omega_{;b}\,\omega_a{}^b+\omega_a{}^b\,dottheta_{;b}-\frac{2}{3}\,\theta_{;b}\,\omega_a{}^b\,\theta+8\,\omega_a{}^b\,\omega\,\omega_{;b}-\omega_a{}^c\,\omega_c{}^b\,\theta_{;b}=0$$

```
> temp := mudot[-B]=cod(-theta·mu,-b) : T(%);
```

$$mudot_{;b}=-\mu\,\theta_{;b}-\mu\,\theta_{;b} \tag{1.49}$$

```
> temp2 := expand(TEDS(temp, total2)) : T(%);
```

$$-\mu\,P_a{}^b\,\theta_{;b}-\frac{4}{3}\,\theta\,P_a{}^b\,\mu_{;b}+\frac{8}{3}\,\omega\,\theta\,P^2_{-a,\,b}\,\omega_{;b}-8\,\omega\,P^2_{-a,\,b}\,omegadot_{;b}-8\,\omega\,P_a{}^b\,\omega_{;b}\,\omega_a{}^b+\omega_a{}^b\,dottheta_{;b}-\frac{2}{3}\,\theta_{;b}\,\omega_a{}^b\,\theta+8\,\omega_a{}^b\,\omega\,\omega_{;b}-\omega_a{}^c\,\omega_c{}^b\,\theta_{;b}=0 \tag{1.50}$$

```
> temp := omegadot[-B]=cod(-2/3·theta·omega,-b) : T(%);
```

$$omegadot_{;b}=-\frac{2}{3}\,\theta_{;b}\,\omega-\frac{2}{3}\,\theta\,\omega_{;b} \tag{1.51}$$

```
> temp3 := expand(TEDS(temp, temp2)) : T(%);
```

$$-\mu\,P_a{}^b\,\theta_{;b}-\frac{4}{3}\,\theta\,P_a{}^b\,\mu_{;b}+8\,\omega\,\theta\,P^2_{-a,\,b}\,\omega_{;b}+\frac{16}{3}\,\omega^2\,P^2_{-a,\,b}\,\theta_{;b}-8\,\omega\,P_a{}^b\,\omega_{;b}\,\omega_a{}^b+\omega_a{}^b\,dottheta_{;b}-\frac{2}{3}\,\theta_{;b}\,\omega_a{}^b\,\theta+8\,\omega_a{}^b\,\omega\,\omega_{;b}-\omega_a{}^c\,\omega_c{}^b\,\theta_{;b}=0 \tag{1.52}$$

```
> tank := temp3 :
> eq[20] : T(%);
```

$$dottheta+\frac{1}{3}\,\theta^2-2\,\omega^2+\frac{1}{2}\,\mu=0 \tag{1.53}$$

```
> temp := expand(TEDS(dottheta=thetadot, eq[20])) : T(%);
```

$$thetadot+\frac{1}{3}\,\theta^2-2\,\omega^2+\frac{1}{2}\,\mu=0 \tag{1.54}$$

```
> temp2 := temp - thetadot : T(%);
```

$$\frac{1}{3}\,\theta^2-2\,\omega^2+\frac{1}{2}\,\mu=-thetadot \tag{1.55}$$

```
> temp3 := -rhs(temp2) =-lhs(temp2) : T(%);
```

$$thetadot=-\frac{1}{3}\,\theta^2+2\,\omega^2-\frac{1}{2}\,\mu \tag{1.56}$$

```
> temp4 := cod(temp3,-b) : T(%);
```

$$thetadot_{;b}=-\frac{2}{3}\,\theta\,\theta_{;b}+4\,\omega\,\omega_{;b}-\frac{1}{2}\,\mu_{;b} \tag{1.57}$$

```
> temp5 := expand(TEDS(temp4, tank)) : T(%);
```

$$-\mu P_a{}^b \theta_{;b} - \frac{4}{3}\theta P_a{}^b \mu_{;b} + 8\omega\theta P^2_{-a,b}\omega_{;b} + \frac{16}{3}\omega^2 P^2_{-a,b}\theta_{;b} - 8\omega P_a{}^b \omega_{;b}\omega_a{}^b + \omega_a{}^b dottheta_{;b} - \frac{2}{3}\theta_{;b}\omega_a{}^b\theta + 8\omega_a{}^b\omega\omega_{;b} - \omega_a{}^c\omega_c{}^b\theta_{;b} = 0 \quad \textbf{(1.58)}$$

\>

\> *temp6* := *expand*$\left(\frac{5}{3}\cdot \text{theta}\cdot eq[40]\right)$: *T*(%);

$$-\frac{40}{3}\omega\theta P_a{}^b\omega_{;b} + \frac{5}{3}\theta P_a{}^b\mu_{;b} + \frac{5}{3}\theta_{;b}\omega_a{}^b\theta = 0 \quad \textbf{(1.59)}$$

\> *temp7* := *temp5* + *temp6* : *T*(%);

$$-\mu P_a{}^b \theta_{;b} + \frac{1}{3}\theta P_a{}^b \mu_{;b} + 8\omega\theta P^2_{-a,b}\omega_{;b} + \frac{16}{3}\omega^2 P^2_{-a,b}\theta_{;b} - 8\omega P_a{}^b \omega_{;b}\omega_a{}^b + \omega_a{}^b dottheta_{;b} + \theta_{;b}\omega_a{}^b\theta + 8\omega_a{}^b\omega\omega_{;b} - \omega_a{}^c\omega_c{}^b\theta_{;b} - \frac{40}{3}\omega\theta P_a{}^b\omega_{;b} = 0 \quad \textbf{(1.60)}$$

\> *temp8* := $\frac{1}{2}\cdot eq[41]$: *T*(%);

$$-4\omega_a{}^b\omega\omega_{;b} + \frac{1}{2}\omega_a{}^c\omega_c{}^b\theta_{;b} + \frac{1}{2}\mu_{;b}\omega_a{}^b = 0 \quad \textbf{(1.61)}$$

\> *total* := *temp7* + *temp8* : *T*(%);

$$-\mu P_a{}^b \theta_{;b} + \frac{1}{3}\theta P_a{}^b \mu_{;b} + 8\omega\theta P^2_{-a,b}\omega_{;b} + \frac{16}{3}\omega^2 P^2_{-a,b}\theta_{;b} - 8\omega P_a{}^b \omega_{;b}\omega_a{}^b + \omega_a{}^b dottheta_{;b} + \theta_{;b}\omega_a{}^b\theta + 4\omega_a{}^b\omega\omega_{;b} - \frac{1}{2}\omega_a{}^c\omega_c{}^b\theta_{;b} - \frac{40}{3}\omega\theta P_a{}^b\omega_{;b} + \frac{1}{2}\mu_{;b}\omega_a{}^b = 0 \quad \textbf{(1.62)}$$

\> *eq*[42] := *collect*(*total*, [*P*[−*a*, *b*], theta[−*B*]], \`*distributed*\`) : *T*(%);

$$\omega_a{}^b dottheta_{;b} + 4\omega_a{}^b\omega\omega_{;b} + \frac{1}{2}\mu_{;b}\omega_a{}^b + 8\omega\theta P^2_{-a,b}\omega_{;b} - \mu P_a{}^b\theta_{;b} + \frac{16}{3}\omega^2 P^2_{-a,b}\theta_{;b} + \left(\omega_a{}^b\theta - \frac{1}{2}\omega_a{}^c\omega_c{}^b\right)\theta_{;b} + \left(\frac{1}{3}\theta\mu_{;b} - 8\omega_a{}^b\omega\omega_{;b} - \frac{40}{3}\omega\theta\omega_{;b}\right)P_a{}^b = 0 \quad \textbf{(1.63)}$$

proof: completed
****

\> *PrintSubArray*(*eq*, 1, 42, *y*);

$$1, T_{ab} = \rho u_a u_b$$

$$2, P_{ab} = u u_{ab} + g_{ab}$$

$$3, P^{a}{}_{b} u^{b} = 0$$

$$4, dX^{a} = u^{b} X^{a}{}_{;b}$$

$$5, du^{a} = u^{b} u^{a}{}_{;b}$$

$$6, u_{a;b} = \frac{1}{3}\, \theta P_{ab} + \sigma_{ab} + \omega_{ab} - du_{a} u_{b}$$

$$7, \theta = u^{a}{}_{;a}$$

$$8, \sigma_{ab} = \frac{1}{2}\, P_{a}{}^{c} P_{b}{}^{d} u_{c;d} + \frac{1}{2}\, P_{b}{}^{c} P_{a}{}^{d} u_{c;d} - \frac{1}{3}\, \theta P_{ab}$$

$$9, \omega_{ab} = \frac{1}{2}\, P_{a}{}^{c} P_{b}{}^{d} u_{c;d} - \frac{1}{2}\, P_{b}{}^{c} P_{a}{}^{d} u_{c;d}$$

$$10, \omega^{a} = \frac{1}{2}\, \eta^{abcd} u_{b} \omega_{cd}$$

$$11, \omega_{ab} = \eta_{abef} \omega^{e} u^{f}$$

$$12, \omega^{2} = \frac{1}{2}\, \omega^{ab} \omega_{ab}$$

13, "iff(iff(omega[-a,-b] = 0,omega[-a]),omega = 0)"

$$14, \omega_{a}{}^{c} \omega_{c}{}^{b} = -\omega^{2} P_{a}{}^{b} + \omega^{b} \omega_{a}$$

$$15, \frac{1}{2}\, u_{b;a} - \frac{1}{2}\, u_{a;b} = \frac{1}{2}\, du_{a} u_{b} - \frac{1}{2}\, du_{b} u_{a} + \omega^{ab}$$

$$16, -\frac{1}{6}\, u_{c} u_{a;b} + \frac{1}{6}\, u_{c} u_{b;a} + \frac{1}{6}\, u_{b} u_{a;c} - \frac{1}{6}\, u_{b} u_{c;a} - \frac{1}{6}\, u_{a} u_{b;c} + \frac{1}{6}\, u_{a} u_{c;b} = 0$$

$$17, \sigma_{ab} = 0$$

$$18, u_{a;b} = \frac{1}{3}\, \theta P_{ab} + \omega_{ab}$$

$$19, u^{a}{}_{;c;d} - u^{a}{}_{;d;c} = R^{a}{}_{bcd} u^{b}$$

$$20, dottheta + \frac{1}{3}\, \theta^{2} - 2\, \omega^{2} + \frac{1}{2}\, \mu = 0$$

$$21, P_{a}{}^{c} P_{b}{}^{d} \omega_{cd;f} u^{f} + \frac{2}{3}\, \theta\, \omega_{ab} = 0$$

$$22, \omega_{a} \omega_{b} - \frac{1}{3}\, P_{ab} \omega^{2} + E_{ab} = 0$$

$$23, E_{ab} = C_{abcd} u^{c} u^{d}$$

$$24, H_{ab} = \frac{1}{2}\, \eta_{ae}{}^{cd} C_{cdbf} u^{e} u^{f}$$

$$25,\ P^{a}{}_{b}\,\omega^{b}{}_{;f}\,u^{f}+\frac{2}{3}\,\theta\,\omega^{a}=0$$

$$26,\ 2\,P^{a\ b}\,\theta_{;b}+3\,P^{a}{}_{b}\,\omega^{b\ d}{}_{;d}=0$$

$$27,\ \omega^{a}{}_{;a}=2\,du^{a}\,\omega_{a}$$

$$28,\ H_{a\,b}=\frac{1}{2}\,P_{a}{}^{c}\,P_{b}{}^{d}\,\omega^{d\ ;c}+\frac{1}{2}\,P_{b}{}^{c}\,P_{a}{}^{d}\,\omega^{d\ ;c}$$

$$29,\ \omega_{a\,b}\,\omega^{b\ c}{}_{;c}=P_{a}{}^{b}\,\omega^{c}\,\omega_{b\,;c}-P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b}$$

$$30,\ \mu\,\theta+dotmu=0$$

$$31,\ (\mu+p)\ du^{a}+P^{a\ b}\,p_{;b}$$

$$32,\ du^{a}=0$$

$$33,\ u_{a}=-\frac{f_{;a}}{fdot}$$

$$34,\ \mu=(c1-1)\,p+c2\,\omega^{2}$$

$$35,\ dotomega_{a\,b}=-\frac{2}{3}\,\theta\,\omega_{a\,b}$$

$$36,\ dotomega=-\frac{2}{3}\,\theta\,\omega$$

$$37,\ \theta\left(c1p-\frac{1}{3}\,c2\,\omega^{2}\right)=0$$

$$38,\ \frac{\partial}{\partial t}\left(P^{a\ b}f_{;b}\right)=P^{a\ b}f^{`\prime`}{}_{;b}+\omega^{a\ b}f_{;b}-\frac{1}{3}\,\theta\,P^{a\ b}f_{;b}$$

$$39,\ -3\,P_{a}{}^{b}\,\omega^{c}\,\omega_{b\,;c}-13\,P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b}+2\,P_{a}{}^{b}\,\mu_{;b}=0$$

$$40,\ -8\,\omega\,P_{a}{}^{b}\,\omega_{;b}+P_{a}{}^{b}\,\mu_{;b}+\omega_{a}{}^{b}\,\theta_{;b}=0$$

$$41,\ -8\,\omega_{a}{}^{b}\,\omega\,\omega_{;b}+\omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}+\mu_{;b}\,\omega_{a}{}^{b}=0$$

$$42,\ \omega_{a}{}^{b}\,dottheta_{;b}+4\,\omega_{a}{}^{b}\,\omega\,\omega_{;b}+\frac{1}{2}\,\mu_{;b}\,\omega_{a}{}^{b}+8\,\omega\,\theta\,P^{2}_{-a,\,b}\,\omega_{;b}-\mu\,P_{a}{}^{b}\,\theta_{;b}+\frac{16}{3}\,\omega^{2}\,P^{2}_{-a,\,b}\,\theta_{;b}+\left(\omega_{a}{}^{b}\,\theta-\frac{1}{2}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b}\right)\theta_{;b}+\left(\frac{1}{3}\,\theta\,\mu_{;b}-8\,\omega_{a}{}^{b}\,\omega\,\omega_{;b}-\frac{40}{3}\,\omega\,\theta\,\omega_{;b}\right)P_{a}{}^{b}=0 \qquad \textbf{(1.64)}$$

```
> save eq, "Seneqs3d";
>
>
```