```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index);
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for dust
page 3

if $\sigma_{a b}$=0 =>ωΘ=0
Author: Peter Huf
file 3- eq41b

In this file we continue to follow the equations outlined by Senovilla et al. (2007) with the assumptions for dust
i.e

```
> read "EFE" : read "SFE" : read "fids" : read "eqs" : read "eqs2" :read "Seneqs3b" :
>
**************************************
Equation 41b
**************************************
>
Proof of equation 41b:
>
>
> eq[41] := -8*omega[-a,b]*omega*omega[-B] + omega[-a,b]*mu[-B] + omega[-a,
      c]*omega[-c,b]*theta[-B] = 0 : T(%);
```

$$-8\,\omega\,\omega_{;b}\,\omega_{a}{}^{b} + \omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b} + \mu_{;b}\,\omega_{a}{}^{b} = 0 \qquad (1.1)$$

Contracting eq41 with $\omega^{c\,a}$ leads to :

```
> temp := expand(eq[41]·omega[-d,a]) : T(%);
```

$$-8\,\omega\,\omega_{;b}\,\omega_{a}{}^{b}\,\omega_{d}{}^{a} + \omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\omega_{d}{}^{a}\,\theta_{;b} + \mu_{;b}\,\omega_{a}{}^{b}\,\omega_{d}{}^{a} = 0 \qquad (1.2)$$

```
> temp2 := subs(c=e, a=c, d=a, e=d, temp) : T(%);
```

$$-8\,\omega\,\omega_{;b}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b} + \omega_{a}{}^{c}\,\omega_{c}{}^{d}\,\omega_{d}{}^{b}\,\theta_{;b} + \mu_{;b}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b} = 0 \qquad (1.3)$$

```
> temp3 := expand(TEDS(eq[14 b], temp2)) : T(%);
```

$$-\theta_{;b}\,\omega^{2}\,\omega_{a}{}^{b} - 8\,\omega\,\omega_{;b}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b} + \mu_{;b}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b} = 0 \qquad (1.4)$$

> $eq[41\ b] := temp3 : T(\%);$

$$-\theta_{;b}\,\omega^{2}\,\omega_{a}{}^{b} - 8\,\omega\,\omega_{;b}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b} + \mu_{;b}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b} = 0 \qquad \textbf{(1.5)}$$

> $eq[41\ c] := temp2 : T(\%);$

$$-8\,\omega\,\omega_{;b}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b} + \omega_{a}{}^{c}\,\omega_{c}{}^{d}\,\omega_{d}{}^{b}\,\theta_{;b} + \mu_{;b}\,\omega_{a}{}^{c}\,\omega_{c}{}^{b} = 0 \qquad \textbf{(1.6)}$$

>

Proof of equation 41b, 41c - completed:

****

>

> $PrintSubArray(eq, 1, 41, y);$

$$1,\ T_{a\,b} = \rho\,u_{a}\,u_{b}$$

$$2,\ P_{a\,b} = u\,u_{a\,b} + g_{a\,b}$$

$$3,\ P^{a}{}_{b}\,u^{b} = 0$$

$$4,\ dX^{a} = u^{b}\,X^{a}{}_{;b}$$

$$5,\ du^{a} = u^{b}\,u^{a}{}_{;b}$$

$$6,\ u_{a\,;b} = \frac{1}{3}\,\theta\,P_{a\,b} + \sigma_{a\,b} + \omega_{a\,b} - du_{a}\,u_{b}$$

$$7,\ \theta = u^{a}{}_{;a}$$

$$8,\ \sigma_{a\,b} = \frac{1}{2}\,P_{a}{}^{c}\,P_{b}{}^{d}\,u_{c\,;d} + \frac{1}{2}\,P_{b}{}^{c}\,P_{a}{}^{d}\,u_{c\,;d} - \frac{1}{3}\,\theta\,P_{a\,b}$$

$$9,\ \omega_{a\,b} = \frac{1}{2}\,P_{a}{}^{c}\,P_{b}{}^{d}\,u_{c\,;d} - \frac{1}{2}\,P_{b}{}^{c}\,P_{a}{}^{d}\,u_{c\,;d}$$

$$10,\ \omega^{a} = \frac{1}{2}\,\eta^{a\,b\,c\,d}\,u_{b}\,\omega_{c\,d}$$

$$11,\ \omega_{a\,b} = \eta_{a\,b\,e\,f}\,\omega^{e}\,u^{f}$$

$$12,\ \omega^{2} = \frac{1}{2}\,\omega^{a\,b}\,\omega_{a\,b}$$

13, "iff(iff(omega[-a,-b] = 0,omega[-a]),omega = 0)"

$$14,\ \omega_{a}{}^{c}\,\omega_{c}{}^{b} = -\omega^{2}\,P_{a}{}^{b} + \omega^{b}\,\omega_{a}$$

$$15,\ \frac{1}{2}\,u_{b\,;a} - \frac{1}{2}\,u_{a\,;b} = \frac{1}{2}\,du_{a}\,u_{b} - \frac{1}{2}\,du_{b}\,u_{a} + \omega^{a\,b}$$

$$16,\ -\frac{1}{6}\,u_{c}\,u_{a\,;b} + \frac{1}{6}\,u_{c}\,u_{b\,;a} + \frac{1}{6}\,u_{b}\,u_{a\,;c} - \frac{1}{6}\,u_{b}\,u_{c\,;a} - \frac{1}{6}\,u_{a}\,u_{b\,;c} + \frac{1}{6}\,u_{a}\,u_{c\,;b} = 0$$

$$17,\ \sigma_{a\,b} = 0$$

$$18,\ u_{a\,;b} = \frac{1}{3}\,\theta\,P_{a\,b} + \omega_{a\,b}$$

$$19,\ u^{a}{}_{;c\,;d} - u^{a}{}_{;d\,;c} = R^{a}{}_{b\,c\,d}\,u^{b}$$

$$20,\ dottheta + \frac{1}{3}\,\theta^{2} - 2\,\omega^{2} + \frac{1}{2}\,\mu = 0$$

$$21,\ P_{a}{}^{c}\,P_{b}{}^{d}\,\omega_{c\,d\,;f}\,u^{f} + \frac{2}{3}\,\theta\,\omega_{a\,b} = 0$$

$$22,\ \omega_{a}\,\omega_{b} - \frac{1}{3}\,P_{a\,b}\,\omega^{2} + E_{a\,b} = 0$$

$$23,\ E_{a\,b} = C_{a\,b\,c\,d}\,u^{c}\,u^{d}$$

$$24,\ H_{a\,b} = \frac{1}{2}\,\eta_{a\,e}{}^{c\,d}\,C_{c\,d\,b\,f}\,u^{e}\,u^{f}$$

$$25,\ P^{a}{}_{b}\,\omega^{b}{}_{;f}\,u^{f} + \frac{2}{3}\,\theta\,\omega^{a} = 0$$

$$26,\ 2\,P^{a\,b}\,\theta_{;b} + 3\,P^{a}{}_{b}\,\omega^{b\,d}{}_{;d} = 0$$

$$27,\ \omega^{a}{}_{;a} = 2\,du^{a}\,\omega_{a}$$

$$28,\ H_{a\,b} = \frac{1}{2}\,P_{a}{}^{c}\,P_{b}{}^{d}\,\omega^{d\,;c} + \frac{1}{2}\,P_{b}{}^{c}\,P_{a}{}^{d}\,\omega^{d\,;c}$$

$$29,\ \omega_{a\,b}\,\omega^{b\,c}{}_{;c} = P_{a}{}^{b}\,\omega^{c}\,\omega_{b\,;c} - P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b}$$

$$30,\ \mu\,\theta + dotmu = 0$$

$$31,\ (\mu + p)\,du^{a} + P^{a\,b}\,p_{;b}$$

$$32,\ du^{a} = 0$$

$$33,\ u_{a} = -\frac{f_{;a}}{fdot}$$

$$34,\ \mu = (c1 - 1)\,p + c2\,\omega^{2}$$

$$35,\ dotomega_{a\,b} = -\frac{2}{3}\,\theta\,\omega_{a\,b}$$

$$36,\ dotomega = -\frac{2}{3}\,\theta\,\omega$$

$$37,\ \theta\left(c1p - \frac{1}{3}\,c2\,\omega^{2}\right) = 0$$

$$38,\ \frac{\partial}{\partial t}\left(P^{a\,b}\,f_{;b}\right) = P^{a\,b}\,fdot_{;b} + \omega^{a\,b}\,f_{;b} - \frac{1}{3}\,\theta\,P^{a\,b}\,f_{;b}$$

$$39,\ -3\,P_{a}{}^{b}\,\omega^{c}\,\omega_{b\,;c} - 13\,P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b} + 2\,P_{a}{}^{b}\,\mu_{;b} = 0$$

$$40,\ -8\,\omega\,P_{a}{}^{b}\,\omega_{;b} + P_{a}{}^{b}\,\mu_{;b} + \omega_{a}{}^{b}\,\theta_{;b} = 0$$

$$41,\ -8\,\omega\,\omega_{;b}\,\omega_{a}{}^{b}+\omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}+\mu_{;b}\,\omega_{a}{}^{b}=0 \qquad \textbf{(1.7)}$$

```
> save eq, "Seneqs3c";
> read "Seneqs3c" :
>
>
```