```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index);
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for dust
page 3

if $\sigma_{a\,b}$=0 =>ωΘ=0
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file 3-eq41

In this file we continue to follow the equations outlined by Senovilla et al. (2007) with the assumptions for dust
i.e

```
> read "EFE" : read "SFE" :read "fids" :read "eqs2" :read "Seneqs3a" :

>
**************************************
Equation 41
**************************************
>
```

Proof of equation 41:

```
> eq[41] := -8*omega[-a,b]*omega*omega[-B] + omega[-a,b]*mu[-B] + omega[-a,
      c]*omega[-c,b]*theta[-B] = 0 : T(%);
```

$$-8\,\omega_{a}{}^{b}\,\omega\,\omega_{;b} + \omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b} + \omega_{a}{}^{b}\,\mu_{;b} = 0 \qquad \textbf{(1.1)}$$

```
> eq[40] := P[-a,b]·mu[-B] - 8·omega·P[-a,b]·omega[-B] + omega[-a,b]·theta[-B]
      = 0 : T(%);
```

$$-8\,\omega\,P_{a}{}^{b}\,\omega_{;b} + P_{a}{}^{b}\,\mu_{;b} + \omega_{a}{}^{b}\,\theta_{;b} = 0 \qquad \textbf{(1.2)}$$

Contracting eq40 with $\omega^{c\,a}$ leads to :

```
> temp := expand(eq[40]·omega[c,a]) : T(%);
```

$$-8\,\omega\,P_{a}{}^{b}\,\omega_{;b}\,\omega^{c\,a} + P_{a}{}^{b}\,\mu_{;b}\,\omega^{c\,a} + \omega^{c\,a}\,\omega_{a}{}^{b}\,\theta_{;b} = 0 \qquad \textbf{(1.3)}$$

```
> temp1 := TEDS(P[-a,b] = g[-a,b] + u[-a]·u[b], temp) : T(%);
```

$$-8\,\omega\,\omega_{;b}\,\omega^{c\,a}\,u^{b}\,u_{a} - 8\,\omega\,g_{a}{}^{b}\,\omega_{;b}\,\omega^{c\,a} + \mu_{;b}\,\omega^{c\,a}\,u^{b}\,u_{a} + g_{a}{}^{b}\,\mu_{;b}\,\omega^{c\,a} + \omega^{c\,a}\,\omega_{a}{}^{b}\,\theta_{;b} = 0 \qquad \textbf{(1.4)}$$

```
> temp2 := Absorbg(temp1) : T(%);
```

0, "not a tensor"

(1.5)

$$-8\,\omega\,\omega_{;b}\,\omega^{c\,a}u^{b}u_{a}+\mu_{;b}\,\omega^{c\,a}u^{b}u_{a}-8\,\omega\,\omega_{;b}\,\omega^{c\,b}+\omega^{c\,a}\omega_{a}{}^{b}\theta_{;b}+\mu_{;b}\,\omega^{c\,b}=0 \tag{1.5}$$

```
> temp3 := TEDS(omega[c, a]·u[-a] = 0, temp2) : T(%);
```

$$-8\,\omega\,\omega_{;b}\,\omega^{c\,b}+\omega^{c\,a}\omega_{a}{}^{b}\theta_{;b}+\mu_{;b}\,\omega^{c\,b}=0 \tag{1.6}$$

```
> eq[41] := subs(a = e, c = -a, e = c, temp3) : T(%);
```

$$-8\,\omega_{a}{}^{b}\,\omega\,\omega_{;b}+\omega_{a}{}^{c}\omega_{c}{}^{b}\theta_{;b}+\omega_{a}{}^{b}\mu_{;b}=0 \tag{1.7}$$

Proof of equation 41 - completed:

****

```
>
```

```
> PrintSubArray(eq, 1, 41, y);
```

$$1,\ T_{a\,b}=\rho\,u_{a}u_{b}$$

$$2,\ P_{a\,b}=u\,u_{b\,a}+g_{a\,b}$$

$$3,\ P^{a}{}_{b}u^{b}=0$$

$$4,\ dX^{a}=u^{b}X^{a}{}_{;b}$$

$$5,\ du^{a}=u^{b}u^{a}{}_{;b}$$

$$6,\ u_{a\,;b}=\frac{1}{3}\,\theta\,P_{a\,b}+\sigma_{a\,b}+\omega_{a\,b}-du_{a}u_{b}$$

$$7,\ \theta=u^{a}{}_{;a}$$

$$8,\ \sigma_{a\,b}=\frac{1}{2}\,P_{a}{}^{c}P_{b}{}^{d}u_{c\,;d}+\frac{1}{2}\,P_{b}{}^{c}P_{a}{}^{d}u_{c\,;d}-\frac{1}{3}\,\theta\,P_{a\,b}$$

$$9,\ \omega_{a\,b}=\frac{1}{2}\,P_{a}{}^{c}P_{b}{}^{d}u_{c\,;d}-\frac{1}{2}\,P_{b}{}^{c}P_{a}{}^{d}u_{c\,;d}$$

$$10,\ \omega^{a}=\frac{1}{2}\,\eta^{a\,b\,c\,d}u_{b}\omega_{c\,d}$$

$$11,\ \omega_{a\,b}=\eta_{a\,b\,e\,f}\omega^{e}u^{f}$$

$$12,\ \omega^{2}=\frac{1}{2}\,\omega^{a\,b}\omega_{a\,b}$$

13, "iff(iff(omega[-a,-b] = 0,omega[-a]),omega = 0)"

$$14,\ \omega_{a}{}^{c}\omega_{c}{}^{b}=-\omega^{2}P_{a}{}^{b}+\omega^{b}\omega_{a}$$

$$15,\ \frac{1}{2}\,u_{b\,;a}-\frac{1}{2}\,u_{a\,;b}=\frac{1}{2}\,du_{a}u_{b}-\frac{1}{2}\,du_{b}u_{a}+\omega^{a\,b}$$

$$16,\ -\frac{1}{6}\,u_{c}u_{a\,;b}+\frac{1}{6}\,u_{c}u_{b\,;a}+\frac{1}{6}\,u_{b}u_{a\,;c}-\frac{1}{6}\,u_{b}u_{c\,;a}-\frac{1}{6}\,u_{a}u_{b\,;c}+\frac{1}{6}\,u_{a}u_{c\,;b}=0$$

$$17,\ \sigma_{a\,b} = 0$$

$$18,\ u_{a\,;b} = \frac{1}{3}\,\theta\,P_{a\,b} + \omega_{a\,b}$$

$$19,\ u^{a}{}_{;c\,;d} - u^{a}{}_{;d\,;c} = R^{a}{}_{b\,c\,d}\,u^{b}$$

$$20,\ dottheta + \frac{1}{3}\,\theta^{2} - 2\,\omega^{2} + \frac{1}{2}\,\mu = 0$$

$$21,\ P_{a}{}^{c}\,P_{b}{}^{d}\,\omega_{c\,d\,;f}\,u^{f} + \frac{2}{3}\,\theta\,\omega_{a\,b} = 0$$

$$22,\ \omega_{a}\,\omega_{b} - \frac{1}{3}\,P_{a\,b}\,\omega^{2} + E_{a\,b} = 0$$

$$23,\ E_{a\,b} = C_{a\,b\,c\,d}\,u^{c}\,u^{d}$$

$$24,\ H_{a\,b} = \frac{1}{2}\,\eta_{a\,e}{}^{c\,d}\,C_{c\,d\,b\,f}\,u^{e}\,u^{f}$$

$$25,\ P^{a}{}_{b}\,\omega^{b}{}_{;f}\,u^{f} + \frac{2}{3}\,\theta\,\omega^{a} = 0$$

$$26,\ 2\,P^{a\,b}\,\theta_{;b} + 3\,P^{a}{}_{b}\,\omega^{b\,d}{}_{;d} = 0$$

$$27,\ \omega^{a}{}_{;a} = 2\,du^{a}\,\omega_{a}$$

$$28,\ H_{a\,b} = \frac{1}{2}\,P_{a}{}^{c}\,P_{b}{}^{d}\,\omega^{d\,;c} + \frac{1}{2}\,P_{b}{}^{c}\,P_{a}{}^{d}\,\omega^{d\,;c}$$

$$29,\ \omega_{a\,b}\,\omega^{b\,c}{}_{;c} = P_{a}{}^{b}\,\omega^{c}\,\omega_{b\,;c} - P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b}$$

$$30,\ \mu\,\theta + dotmu = 0$$

$$31,\ (\mu + p)\,du^{a} + P^{a\,b}\,p_{;b}$$

$$32,\ du^{a} = 0$$

$$33,\ u_{a} = -\frac{f_{;a}}{fdot}$$

$$34,\ \mu = (c1 - 1)\,p + c2\,\omega^{2}$$

$$35,\ dotomega_{a\,b} = -\frac{2}{3}\,\theta\,\omega_{a\,b}$$

$$36,\ dotomega = -\frac{2}{3}\,\theta\,\omega$$

$$37,\ \theta\left(c1p - \frac{1}{3}\,c2\,\omega^{2}\right) = 0$$

$$38,\ \frac{\partial}{\partial t}\left(P^{a\,b}\,f_{;b}\right) = P^{a\,b}\,fdot_{;b} + \omega^{a\,b}\,f_{;b} - \frac{1}{3}\,\theta\,P^{a\,b}\,f_{;b}$$

$$39,\ -3\,P_{a}{}^{b}\,\omega^{c}\,\omega_{b\,;c} - 13\,P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b} + 2\,P_{a}{}^{b}\,\mu_{;b} = 0$$

$$40,\ -8\,\omega\,P_{a}{}^{b}\,\omega_{;b}+P_{a}{}^{b}\,\mu_{;b}+\omega_{a}{}^{b}\,\theta_{;b}=0$$

$$41,\ -8\,\omega_{a}{}^{b}\,\omega\,\omega_{;b}+\omega_{a}{}^{c}\,\omega_{c}{}^{b}\,\theta_{;b}+\omega_{a}{}^{b}\,\mu_{;b}=0 \qquad \textbf{(1.8)}$$

```
> save eq, "Seneqs3b";
> read "Seneqs3b" :
>
```