> restart;

> with(Riemann):with(Canon):

> $with(TensorPack) : CDF(0) : CDS(index);$

**Chapter XX**
**Tensor analysis using indices - Senovilla et al. - Shearfree for dust**
**page 3**

**if $\sigma_{a\,b}$=0 =>ωΘ=0**
**Author: Peter Huf**
**file 3a-eq40**

In this file we continue to follow the equations outlined by Senovilla et al. (2007) with the assumptions for dust
i.e

> **read** "EFE" : **read** "SFE" :**read** "fids" :**read** "eqs2" :**read** "Seneqs3" :

>

**Equation 40**

*************************************

**Proof of equation 40:**

Using eq26 contracted with $\omega_{a\,c}$ and the identity eq29, the equation 39 can be rewritten as :

> $eq[40] := P[-a, b]\cdot mu[-B] - 8\cdot omega\cdot P[-a, b]\cdot omega[-B] + omega[-a, b]\cdot theta[-B] = 0 : T(\%);$

$$-8\,\omega\,P_a{}^b\,\omega_{;b} + P_a{}^b\,\mu_{;b} + \omega_a{}^b\,\theta_{;b} = 0 \qquad \textbf{(1.1)}$$

> $eq[39] := 2\cdot P[-a, b]\cdot mu[-B] - 13\cdot P[-a, b]\cdot omega[c]\cdot omega[-c, -B] - 3\cdot P[-a, b]\cdot omega[c]\cdot omega[-b, -C] = 0 : T(\%);$

$$-3\,P_a{}^b\,\omega^c\,\omega_{b\,;c} - 13\,P_a{}^b\,\omega^c\,\omega_{c\,;b} + 2\,P_a{}^b\,\mu_{;b} = 0 \qquad \textbf{(1.2)}$$

proof :

> $eq[26] := 2\cdot P[a, b]\cdot theta[-B] + 3\cdot P[a, -b]\cdot omega[b, d, -D] = 0 : T(\%);$

$$2\,P^{a\,b}\,\theta_{;b} + 3\,P^a{}_b\,\omega^{b\,d}{}_{;d} = 0 \qquad \textbf{(1.3)}$$

> $eq[29] := omega[-a, -b]\cdot omega[b, c, -C] = P[-a, b]\cdot omega[c]\cdot omega[-b, -C] - P[-a, b]\cdot omega[c]\cdot omega[-c, -B] : T(\%);$

$$\omega_{a\,b}\,\omega^{b\,c}{}_{;c} = P_a{}^b\,\omega^c\,\omega_{b\,;c} - P_a{}^b\,\omega^c\,\omega_{c\,;b} \qquad \textbf{(1.4)}$$

contracting with $\omega_{a\,c}$

> $temp := expand(omega[-a, -c]\cdot eq[26\,]) : T(\%);$

$$2\,P^{a\,b}\,\omega_{a\,c}\,\theta_{;b} + 3\,P^{a}{}_{b}\,\omega_{a\,c}\,\omega^{b\,d}{}_{;d} = 0 \quad (1.5)$$

> temp2 := TEDS(P[a, b] = g[a, b] + u[a]·u[b], *temp*) : *T*(%);

$$2\,\omega_{a\,c}\,\theta_{;b}\,u^{a}\,u^{b} + 3\,P^{a}{}_{b}\,\omega_{a\,c}\,\omega^{b\,d}{}_{;d} + 2\,g^{a\,b}\,\omega_{a\,c}\,\theta_{;b} = 0 \quad (1.6)$$

> temp3 := expand(TEDS(P[a, -b] = g[a, -b] + u[a]·u[-b], *temp2*)) : *T*(%);

$$3\,\omega_{a\,c}\,\omega^{b\,d}{}_{;d}\,u^{a}\,u_{b} + 2\,\omega_{a\,c}\,\theta_{;b}\,u^{a}\,u^{b} + 2\,g^{a\,b}\,\omega_{a\,c}\,\theta_{;b} + 3\,g^{a}{}_{b}\,\omega_{a\,c}\,\omega^{b\,d}{}_{;d} = 0 \quad (1.7)$$

> temp4 := expand(TEDS(omega[-a, -c]·u[a] = 0, *temp3*)) : *T*(%);

$$2\,g^{a\,b}\,\omega_{a\,c}\,\theta_{;b} + 3\,g^{a}{}_{b}\,\omega_{a\,c}\,\omega^{b\,d}{}_{;d} = 0 \quad (1.8)$$

> temp5 := Absorbg(temp4) : *T*(%);

0, "not a tensor"

$$2\,\omega^{b}{}_{c}\,\theta_{;b} + 3\,\omega_{b\,c}\,\omega^{b\,d}{}_{;d} = 0 \quad (1.9)$$

> *temp6* := expand(TEDS(omega[-b, -c] = -omega[-c, -b], *temp5*)) : *T*(%);

$$2\,\omega^{b}{}_{c}\,\theta_{;b} - 3\,\omega^{b\,d}{}_{;d}\,\omega_{c\,b} = 0 \quad (1.10)$$

Now eq29:

> *eq*[29] := omega[-*a*, -*b*]·omega[*b*, *c*, -*C*] = *P*[-*a*, *b*]·omega[*c*]·omega[-*b*, -*C*] − *P*[-*a*, *b*]·omega[-*c*, -*B*]·omega[*c*] : *T*(%);

$$\omega_{a\,b}\,\omega^{b\,c}{}_{;c} = P_{a}{}^{b}\,\omega^{c}\,\omega_{b\,;c} - P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b} \quad (1.11)$$

> *temp* := *subs*(*c* = *d*, *C* = D, *eq*[29]) : *T*(%);

$$\omega_{a\,b}\,\omega^{b\,d}{}_{;d} = P_{a}{}^{b}\,\omega^{d}\,\omega_{b\,;d} - P_{a}{}^{b}\,\omega^{d}\,\omega_{d\,;b} \quad (1.12)$$

> *temp2a* := *subs*(*a* = *c*, *temp*) : *T*(%);

$$\omega^{b\,d}{}_{;d}\,\omega_{c\,b} = P_{c}{}^{b}\,\omega^{d}\,\omega_{b\,;d} - P_{c}{}^{b}\,\omega^{d}\,\omega_{d\,;b} \quad (1.13)$$

> temp3a := expand(TEDS(temp2a, temp6)) : *T*(%);

$$-3\,P_{c}{}^{b}\,\omega^{d}\,\omega_{b\,;d} + 3\,P_{c}{}^{b}\,\omega^{d}\,\omega_{d\,;b} + 2\,\omega^{b}{}_{c}\,\theta_{;b} = 0 \quad (1.14)$$

> *temp4a* := *isolate*(*temp3a*, -3·*P*[-*c*, *b*]·omega[*d*]·omega[-*b*, -D]) : *T*(%);

$$-3\,P_{c}{}^{b}\,\omega^{d}\,\omega_{b\,;d} = -3\,P_{c}{}^{b}\,\omega^{d}\,\omega_{d\,;b} - 2\,\omega^{b}{}_{c}\,\theta_{;b} \quad (1.15)$$

> *temp5a* := -*temp4a* : *T*(%);

$$3\,P_{c}{}^{b}\,\omega^{d}\,\omega_{b\,;d} = 3\,P_{c}{}^{b}\,\omega^{d}\,\omega_{d\,;b} + 2\,\omega^{b}{}_{c}\,\theta_{;b} \quad (1.16)$$

> *temp6a* := *subs*(*c* = *a*, *d* = *c*, D = *C*, *temp5a*) : *T*(%);

$$3\,P_{a}{}^{b}\,\omega^{c}\,\omega_{b\,;c} = 3\,P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b} + 2\,\omega^{b}{}_{a}\,\theta_{;b} \quad (1.17)$$

> $\frac{1}{2}$ · *TEDS*(*temp6a*, *eq*[39]) : *T*(%);

$$-8\,P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b} + P_{a}{}^{b}\,\mu_{;b} - \omega^{b}{}_{a}\,\theta_{;b} = 0 \quad (1.18)$$

```
> eq[40] : T(%);
```

$$-8\,\omega\,P_{a}{}^{b}\,\omega_{;b}+P_{a}{}^{b}\,\mu_{;b}+\omega_{a}{}^{b}\,\theta_{;b}=0 \tag{1.19}$$

proof completed

This is a key equation showing relationships between spatial gradients of density, rotation and expansion

```
> save eq, "Seneqs3a";
> read "Seneqs3a" :
> #PrintSubArray(eq, 1, 40, y);
>
```