```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index) :
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for dust
page 3

if $\sigma_{a\,b}$=0 =>ωΘ=0
Author: Peter Huf
file 3
eq 39- using sopuerta thesis equations

In this file we continue to follow the equations outlined by Senovilla et al. (2007) with the assumptions for dust. In particular this file examines the equation 39 with the various terms involved in the time differentiation of eq26 to eq39
i.e

```
> read "EFE" : read "SFE" :read "fids" :read "eqs2" :read "Seneqs2f" : read "vids" :
>
```

****

****

Equation 39

****

Proof of eq39:

```
> eq[39] := 2·P[-a, b]·mu[-B] − 13·P[-a, b]·omega[c]·omega[-c,-B] − 3·P[-a, b]
    ·omega[c]·omega[-b,-C] = 0 : T(%);
```

$$-3\,P_{a}^{\;\;b}\,\omega^{c}\,\omega_{b\,;c}-13\,P_{a}^{\;\;b}\,\omega^{c}\,\omega_{c\,;b}+2\,P_{a}^{\;\;b}\,\mu_{;b}=0 \qquad \textbf{(1.1)}$$

The first step involves a time derivative of eq26 using eq38:

```
> eq[26] := 2· P[a, b]·theta[-B] + 3·P[a,-b]·omega[b, d,-D] = 0 : T(%);
```

$$2\,P^{a\,b}\,\theta_{;b}+3\,P^{a}_{\;\;b}\,\omega^{b\,d}_{\;\;\;\;;d}=0 \qquad \textbf{(1.2)}$$

We start with identities for the magnetic and electric compnents of the Weyl tensor, for dust (for references see Sopuerta thesis, or Ellis 1970)

```
> sop[3.18] := H[-a,-b] = 1/2 ·P[-a, c]·P[-b, d]·(omega[-d,-C] + omega[-c,-D]) :
    T(%);
```

$$H_{ab} = \frac{1}{2} P_a{}^c P_b{}^d \left(\omega_{c;d} + \omega_{d;c}\right) \tag{1.3}$$

> sop[3.17] := E[-a,-b] = 1/3 · P[-a,-b]·omega·omega − omega[-a]·omega[-b] : T(%);

$$E_{ab} = \frac{1}{3} P_{ab} \omega^2 - \omega_a \omega_b \tag{1.4}$$

Furthermore, eq 2.107 of Sopuerta 1996 (thesis), for dust:

> sop[2.107] := P[-a, c]·P[-b, d]·E[-c,-d, B] + 3·H[-a,-b]·omega[b] = 1/3 ·P[-a, b]
·mu[-B] : T(%);

$$P_a{}^c P_b{}^d E_{cd}{}^{;b} + 3 H_{ab} \omega^b = \frac{1}{3} P_a{}^b \mu_{;b} \tag{1.5}$$

>

> temp1 := subs(a = c, b = d, sop[3.17]) : T(%);

$$E_{cd} = \frac{1}{3} P_{cd} \omega^2 - \omega_c \omega_d \tag{1.6}$$

> temp2 := cod(temp1, b) : T(%);

$$E_{cd}{}^{;b} = \frac{1}{3} P_{cd}{}^{;b} \omega^2 + \frac{2}{3} P_{cd} \omega \omega^{;b} - \omega_c \omega_d{}^{;b} - \omega_c{}^{;b} \omega_d \tag{1.7}$$

>

> temp3 := expand(TEDS(temp2, sop[2.107])) : T(%);

$$\frac{1}{3} P_a{}^c P_b{}^d P_{cd}{}^{;b} \omega^2 + \frac{2}{3} P_a{}^c P_b{}^d P_{cd} \omega \omega^{;b} - P_a{}^c P_b{}^d \omega_c \omega_d{}^{;b} - P_a{}^c P_b{}^d \omega_c{}^{;b} \omega_d + 3 H_{ab} \omega^b = \frac{1}{3} P_a{}^b \mu_{;b} \tag{1.8}$$

> temp4 := TEDS(P[-c,-d, B] = 0, temp3) : T(%);

$$\frac{2}{3} P_a{}^c P_b{}^d P_{cd} \omega \omega^{;b} - P_a{}^c P_b{}^d \omega_c \omega_d{}^{;b} - P_a{}^c P_b{}^d \omega_c{}^{;b} \omega_d + 3 H_{ab} \omega^b = \frac{1}{3} P_a{}^b \mu_{;b} \tag{1.9}$$

> temp5 := TEDS(sop[3.18], temp4) : T(%);

$$\frac{2}{3} P_a{}^c P_b{}^d P_{cd} \omega \omega^{;b} - P_a{}^c P_b{}^d \omega_c \omega_d{}^{;b} - P_a{}^c P_b{}^d \omega_c{}^{;b} \omega_d + \frac{3}{2} \omega^b P_a{}^c P_b{}^d \omega_{c;d} + \frac{3}{2} \omega^b P_a{}^c P_b{}^d \omega_{d;c} = \frac{1}{3} P_a{}^b \mu_{;b} \tag{1.10}$$

Now we use the identity

> temp := P[-a, c]·omega[-c] = omega[-a] : T(%);

$$P_a{}^c \omega_c = \omega_a \tag{1.11}$$

> temp6 := expand(TEDS(temp, temp5)) : T(%);

$$\frac{2}{3} P_{a}{}^{c} P_{b}{}^{d} P_{cd} \omega \omega^{;b} - P_{a}{}^{c} P_{b}{}^{d} \omega_{c}{}^{;b} \omega_{d} + \frac{3}{2} \omega^{b} P_{a}{}^{c} P_{b}{}^{d} \omega_{c;d} + \frac{3}{2} \omega^{b} P_{a}{}^{c} P_{b}{}^{d} \omega_{d;c} - P_{b}{}^{d} \omega_{d}{}^{;b} \omega_{a} = \frac{1}{3} P_{a}{}^{b} \mu_{;b} \quad \textbf{(1.12)}$$

```
> temp := P[-a, c]·P[-b, d]·P[-c,-d] = P[-a,-b] : T(%);
```

$$P_{a}{}^{c} P_{b}{}^{d} P_{cd} = P_{ab} \quad \textbf{(1.13)}$$

```
> temp7 := 3· expand(TEDS(temp, temp6)) : T(%);
```

$$-3 P_{a}{}^{c} P_{b}{}^{d} \omega_{c}{}^{;b} \omega_{d} + \frac{9}{2} \omega^{b} P_{a}{}^{c} P_{b}{}^{d} \omega_{c;d} + \frac{9}{2} \omega^{b} P_{a}{}^{c} P_{b}{}^{d} \omega_{d;c} + 2 \omega \omega^{;b} P_{ab} - 3 P_{b}{}^{d} \omega_{d}{}^{;b} \omega_{a} = P_{a}{}^{b} \mu_{;b} \quad \textbf{(1.14)}$$

Now

```
> expr := P[-b, d]·omega[-d, B] : T(%);
```

$$P_{b}{}^{d} \omega_{d}{}^{;b} \quad \textbf{(1.15)}$$

```
> expr2 := Absorbg(expand(TEDS(P[-b, d] = g[-b, d] + u[-b]·u[d], expr))) : T(%);
```

$$\omega_{d}{}^{;b} u^{d} u_{b} + \omega_{d}{}^{;d} \quad \textbf{(1.16)}$$

```
> expr3 := TEDS(omega[-d, D] = 0, expr2) : T(%);
```

$$\omega_{d}{}^{;b} u^{d} u_{b} \quad \textbf{(1.17)}$$

```
> expr4 := TEDS(omega[-d, B]·u[-b] = dotomega[-d], expr3) : T(%);
```

$$u^{d} \mathit{dotomega}_{d} \quad \textbf{(1.18)}$$

```
> expr5 := TEDS(dotomega[-d] = -2/3·theta·omega[-d], expr4) : T(%);
```

$$-\frac{2}{3} u^{d} \theta \omega_{d} \quad \textbf{(1.19)}$$

```
> expr6 := TEDS(dotomega[-d] = -2/3·theta·omega[-d], expr5) : T(%);
```

$$-\frac{2}{3} u^{d} \theta \omega_{d} \quad \textbf{(1.20)}$$

```
> expr7 := TEDS(omega[-d]·u[d] = 0, expr6) : T(%);
```

$$0 \quad \textbf{(1.21)}$$

```
> expr8 := expr = expr7 : T(%);
```

$$P_{b}{}^{d} \omega_{d}{}^{;b} = 0 \quad \textbf{(1.22)}$$

and so we have

```
> temp8 := expand(TEDS(expr8, temp7)) : T(%);
```

$$-3 P_{a}{}^{c} P_{b}{}^{d} \omega_{c}{}^{;b} \omega_{d} + \frac{9}{2} \omega^{b} P_{a}{}^{c} P_{b}{}^{d} \omega_{c;d} + \frac{9}{2} \omega^{b} P_{a}{}^{c} P_{b}{}^{d} \omega_{d;c} \quad \textbf{(1.23)}$$

$$+2\,\omega\,\omega^{;b} P_{ab} = P_a{}^b \mu_{;b}$$

****

Now

> expr := P[-b, d]·omega[-c, B] : T(%);

$$P_b{}^d \omega_c{}^{;b} \tag{1.24}$$

> expr2 := Absorbg(expand(TEDS(P[-b, d] = g[-b, d] + u[-b]·u[d], expr))) : T(%);

$$\omega_c{}^{;b} u^d u_b + \omega_c{}^{;d} \tag{1.25}$$

> expr3 := TEDS(omega[-c, B]·u[-b] = dotomega[-c], expr2) : T(%);

$$u^d\, dotomega_c + \omega_c{}^{;d} \tag{1.26}$$

> expr4 := TEDS$\left(dotomega[-c] = -\frac{2}{3}\cdot\text{theta}\cdot\text{omega}[-c], expr3\right)$: T(%);

$$-\frac{2}{3} u^d \theta\, \omega_c + \omega_c{}^{;d} \tag{1.27}$$

> temp9 := expand(TEDS(expr = expr4, temp8)) : T(%);

$$\frac{9}{2} \omega^b P_a{}^c P_b{}^d \omega_{c;d} + \frac{9}{2} \omega^b P_a{}^c P_b{}^d \omega_{d;c} + 2 P_a{}^c \omega_d u^d \theta\, \omega_c + 2\,\omega\,\omega^{;b} P_{ab} - 3 P_a{}^c \omega_d \omega_c{}^{;d} = P_a{}^b \mu_{;b} \tag{1.28}$$

> temp10 := expand(TEDS(omega[-d]·u[d] = 0, temp9)) : T(%);

$$\frac{9}{2} \omega^b P_a{}^c P_b{}^d \omega_{c;d} + \frac{9}{2} \omega^b P_a{}^c P_b{}^d \omega_{d;c} + 2\,\omega\,\omega^{;b} P_{ab} - 3 P_a{}^c \omega_d \omega_c{}^{;d} = P_a{}^b \mu_{;b} \tag{1.29}$$

****

also

> expr := P[-b, d]·omega[-d, -C] : T(%);

$$P_b{}^d \omega_{d;c} \tag{1.30}$$

> expr2 := Absorbg(expand(TEDS(P[-b, d] = g[-b, d] + u[-b]·u[d], expr))) : T(%);

$$\omega_{d;c} u^d u_b + \omega_{b;c} \tag{1.31}$$

>

> temp11 := expand(TEDS(expr = expr2, temp10)) : T(%);

$$\frac{9}{2} \omega^b P_a{}^c P_b{}^d \omega_{c;d} + \frac{9}{2} \omega^b P_a{}^c u_b u^d \omega_{d;c} + \frac{9}{2} \omega^b P_a{}^c \omega_{b;c} + 2\,\omega\,\omega^{;b} P_{ab} - 3 P_a{}^c \omega_d \omega_c{}^{;d} = P_a{}^b \mu_{;b} \tag{1.32}$$

> temp12 := expand(TEDS(omega[b]·u[-b] = 0, temp11)) : T(%);

$$\frac{9}{2}\,\omega^{b} P_{a}{}^{c} P_{b}{}^{d} \omega_{c;d} + \frac{9}{2}\,\omega^{b} P_{a}{}^{c} \omega_{b;c} + 2\,\omega\,\omega^{;b} P_{ab} - 3\,P_{a}{}^{c} \omega_{d} \omega_{c}{}^{;d} = P_{a}{}^{b} \mu_{;b} \quad \textbf{(1.33)}$$

****

and also

> *expr* := P[-b, d]·omega[-c,-D] : T(%);

$$P_{b}{}^{d} \omega_{c;d} \quad \textbf{(1.34)}$$

> *expr2* := Absorbg(expand(TEDS(P[-b, d]=g[-b, d]+u[-b]·u[d], expr))) : T(%);

$$\omega_{c;d} u^{d} u_{b} + \omega_{c;b} \quad \textbf{(1.35)}$$

> *expr3* := TEDS(omega[-c,-D]·u[d]=dotomega[-c], expr2) : T(%);

$$u\, dotomega_{bc} + \omega_{c;b} \quad \textbf{(1.36)}$$

> *expr4* := TEDS(dotomega[-c]=-$\frac{2}{3}$·theta·omega[-c], expr3) : T(%);

$$-\frac{2}{3}\,u_{b}\,\theta\,\omega_{c} + \omega_{c;b} \quad \textbf{(1.37)}$$

>

> *temp13* := expand(TEDS(expr=expr4, temp12)) : T(%);

$$-3\,\omega^{b} P_{a}{}^{c} u_{b}\,\theta\,\omega_{c} + \frac{9}{2}\,\omega^{b} P_{a}{}^{c} \omega_{c;b} + \frac{9}{2}\,\omega^{b} P_{a}{}^{c} \omega_{b;c} + 2\,\omega\,\omega^{;b} P_{ab} - 3\,P_{a}{}^{c} \omega_{d} \omega_{c}{}^{;d} = P_{a}{}^{b} \mu_{;b} \quad \textbf{(1.38)}$$

> *temp14* := expand(TEDS(omega[b]·u[-b]=0, temp13)) : T(%);

$$\frac{9}{2}\,\omega^{b} P_{a}{}^{c} \omega_{c;b} + \frac{9}{2}\,\omega^{b} P_{a}{}^{c} \omega_{b;c} + 2\,\omega\,\omega^{;b} P_{ab} - 3\,P_{a}{}^{c} \omega_{d} \omega_{c}{}^{;d} = P_{a}{}^{b} \mu_{;b} \quad \textbf{(1.39)}$$

> *temp15* := subs(d=-b, D=-B, temp14) : T(%);

$$\frac{3}{2}\,\omega^{b} P_{a}{}^{c} \omega_{c;b} + \frac{9}{2}\,\omega^{b} P_{a}{}^{c} \omega_{b;c} + 2\,\omega\,\omega^{;b} P_{ab} = P_{a}{}^{b} \mu_{;b} \quad \textbf{(1.40)}$$

> *temp16* := expand(TEDS(2·omega·omega[B]·P[-a,-b]=2·omega[d]·omega[-d,-B]·P[-a, b], temp15)) : T(%);

$$\frac{3}{2}\,\omega^{b} P_{a}{}^{c} \omega_{c;b} + \frac{9}{2}\,\omega^{b} P_{a}{}^{c} \omega_{b;c} + 2\,\omega^{d} \omega_{d;b} P_{a}{}^{b} = P_{a}{}^{b} \mu_{;b} \quad \textbf{(1.41)}$$

> *temp17* := expand(TEDS(omega[d]·omega[-d,-B]·P[-a, b]=omega[b]·omega[-b,-C]·P[-a, c], temp16)) : T(%);

$$\frac{3}{2}\,\omega^{b} P_{a}{}^{c} \omega_{c;b} + \frac{13}{2}\,\omega^{b} P_{a}{}^{c} \omega_{b;c} = P_{a}{}^{b} \mu_{;b} \quad \textbf{(1.42)}$$

> *temp18* := subs(b=e, B=E, c=b, C=B, e=d, E=D, temp17) : T(%);

$$\frac{3}{2}\,\omega^{d} P_{a}{}^{b} \omega_{b;d} + \frac{13}{2}\,\omega^{d} \omega_{d;b} P_{a}{}^{b} = P_{a}{}^{d} \mu_{;d} \quad \textbf{(1.43)}$$

which is eq 39

> eq[39] := 2·P[-a, b]·mu[-B] − 13·P[-a, b]·omega[c]·omega[-c,-B] − 3·P[-a, b]·omega[c]·omega[-b,-C] = 0 : T(%);

$$-3\, P_{a}{}^{b}\, \omega^{c}\, \omega_{b\,;c} - 13\, P_{a}{}^{b}\, \omega^{c}\, \omega_{c\,;b} + 2\, P_{a}{}^{b}\, \mu_{;b} = 0 \qquad \textbf{(1.44)}$$

> PrintSubArray(eq, 1, 39, y);

$$1,\ T_{a\,b} = \rho\, u_{a}\, u_{b}$$

$$2,\ P_{a\,b} = u\, u_{b\,a} + g_{a\,b}$$

$$3,\ P^{a}{}_{b}\, u^{b} = 0$$

$$4,\ dX^{a} = u^{b}\, X^{a}{}_{;b}$$

$$5,\ du^{a} = u^{b}\, u^{a}{}_{;b}$$

$$6,\ u_{a\,;b} = \frac{1}{3}\, \theta\, P_{a\,b} + \sigma_{a\,b} + \omega_{a\,b} - du_{a}\, u_{b}$$

$$7,\ \theta = u^{a}{}_{;a}$$

$$8,\ \sigma_{a\,b} = \frac{1}{2}\, P_{a}{}^{c}\, P_{b}{}^{d}\, u_{c\,;d} + \frac{1}{2}\, P_{b}{}^{c}\, P_{a}{}^{d}\, u_{c\,;d} - \frac{1}{3}\, \theta\, P_{a\,b}$$

$$9,\ \omega_{a\,b} = \frac{1}{2}\, P_{a}{}^{c}\, P_{b}{}^{d}\, u_{c\,;d} - \frac{1}{2}\, P_{b}{}^{c}\, P_{a}{}^{d}\, u_{c\,;d}$$

$$10,\ \omega^{a} = \frac{1}{2}\, \eta^{a\,b\,c\,d}\, u_{b}\, \omega_{c\,d}$$

$$11,\ \omega_{a\,b} = \eta_{a\,b\,e\,f}\, \omega^{e}\, u^{f}$$

$$12,\ \omega^{2} = \frac{1}{2}\, \omega^{a\,b}\, \omega_{a\,b}$$

13, "iff(iff(omega[-a,-b] = 0,omega[-a]),omega = 0)"

$$14,\ \omega_{a}{}^{c}\, \omega_{c}{}^{b} = -\omega^{2}\, P_{a}{}^{b} + \omega^{b}\, \omega_{a}$$

$$15,\ \frac{1}{2}\, u_{b\,;a} - \frac{1}{2}\, u_{a\,;b} = \frac{1}{2}\, du_{a}\, u_{b} - \frac{1}{2}\, du_{b}\, u_{a} + \omega^{a\,b}$$

$$16,\ -\frac{1}{6}\, u_{c}\, u_{a\,;b} + \frac{1}{6}\, u_{c}\, u_{b\,;a} + \frac{1}{6}\, u_{b}\, u_{a\,;c} - \frac{1}{6}\, u_{b}\, u_{c\,;a} - \frac{1}{6}\, u_{a}\, u_{b\,;c} + \frac{1}{6}\, u_{a}\, u_{c\,;b} = 0$$

$$17,\ \sigma_{a\,b} = 0$$

$$18,\ u_{a\,;b} = \frac{1}{3}\, \theta\, P_{a\,b} + \omega_{a\,b}$$

$$19,\ u^{a}{}_{;c\,;d} - u^{a}{}_{;d\,;c} = R^{a}{}_{b\,c\,d}\, u^{b}$$

$$20,\ dottheta + \frac{1}{3}\, \theta^{2} - 2\, \omega^{2} + \frac{1}{2}\, \mu = 0$$

$$21,\ P_{a}{}^{c}\, P_{b}{}^{d}\, \omega_{c\,d\,;f}\, u^{f} + \frac{2}{3}\, \theta\, \omega_{a\,b} = 0$$

$$22, \omega_{a}\omega_{b} - \frac{1}{3} P_{ab}\omega^{2} + E_{ab} = 0$$

$$23, E_{ab} = C_{abcd} u^{c} u^{d}$$

$$24, H_{ab} = \frac{1}{2} \eta_{ae}{}^{cd} C_{cdbf} u^{e} u^{f}$$

$$25, P^{a}{}_{b} \omega^{b}{}_{;f} u^{f} + \frac{2}{3} \theta \omega^{a} = 0$$

$$26, 2 P^{ab} \theta_{;b} + 3 P^{a}{}_{b} \omega^{bd}{}_{;d} = 0$$

$$27, \omega^{a}{}_{;a} = 2\, du^{a} \omega_{a}$$

$$28, H_{ab} = \frac{1}{2} P_{a}{}^{c} P_{b}{}^{d} \omega^{d;c} + \frac{1}{2} P_{b}{}^{c} P_{a}{}^{d} \omega^{d;c}$$

$$29, \omega_{an} \omega^{nm}{}_{;m} = \omega^{2} du_{a} + P_{a}{}^{b} \omega^{c} \omega_{b;c} - P_{a}{}^{b} \omega^{c} \omega_{c;b} - du_{p} \omega^{p} \omega_{a}$$

$$30, \mu\theta + dotmu = 0$$

$$31, (\mu + p)\, du^{a} + P^{ab} p_{;b}$$

$$32, du^{a} = 0$$

$$33, u_{a} = -\frac{f_{;a}}{fdot}$$

$$34, \mu = (c1 - 1)\, p + c2\,\omega^{2}$$

$$35, dotomega_{ab} = -\frac{2}{3} \theta \omega_{ab}$$

$$36, dotomega = -\frac{2}{3} \theta \omega$$

$$37, \theta \left(c1p - \frac{1}{3} c2\,\omega^{2} \right) = 0$$

$$38, \frac{\partial}{\partial t} \left(P^{ab} f_{;b} \right) = P^{ab} fdot_{;b} + \omega^{ab} f_{;b} - \frac{1}{3} \theta P^{ab} f_{;b}$$

$$39, -3 P_{a}{}^{b} \omega^{c} \omega_{b;c} - 13 P_{a}{}^{b} \omega^{c} \omega_{c;b} + 2 P_{a}{}^{b} \mu_{;b} = 0 \tag{1.45}$$

```
> save eq, "Seneqs3" :
> read "Seneqs3" :
>
>
>
```