```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index) :
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for dust
page 2
if $\sigma_{a\,b}$=0 =>ωΘ=0
Author: Peter Huf
file 2e:eqs32-37

In this file we continue to follow the equations outlined by Senovilla et al. (1997).

In this case we introduce the assumption for dust:
a=du=0
p=0
i.e

```
> read "EFE" : read "SFE" :read "fids" :read "Seneqs2e1" :
```

****
Equation 32
****

assumption du=0, is important in simplifying many equations

```
> eq[32] := du[a]=0 : T(%);
```

$$du^{\;a} = 0 \qquad \textbf{(1.1)}$$

```
>
```

****
Lemmas 1-3, including eq35-37
****

Lemmas 1-3 are assumed here
not proven in this context - see paper for proof, which are accepted as given

Lemma 1: If there exists a function f satisfying $P^{a\,b} f_{;a} = 0$ then either f=const or the rotation vanishes.

```
> eq[33] := u[-a]=- 1/fdot ·f[-A] : T(%);
```

$$u_a = -\frac{f_{;a}}{fdot} \quad \textbf{(1.2)}$$

Lemma 2: If the perfect fluid is geodesic, then either the pressure p is constant or the rotation vanishes.

> $eq[34] := \mathrm{mu} = (c1 - 1)\cdot p + c2\cdot \mathrm{omega}\cdot\mathrm{omega} : T(\%);$

$$\mu = (c1 - 1)\, p + c2\, \omega^2 \quad \textbf{(1.3)}$$

Lemma 3: If the perfect fluid is geodesic and shear-free, and there exist constants c1 and c2, (c2<>0) such that $\mu = (c1 - 1)\, p + c2\, \omega^2$, then either the rotation or expansion vanishes.

> $eq[35] := dotomega[-a,-b] = -\left(\frac{2}{3}\right)\cdot\mathrm{theta}\cdot\mathrm{omega}[-a,-b] : T(\%);$

$$dotomega_{a\,b} = -\frac{2}{3}\,\theta\,\omega_{a\,b} \quad \textbf{(1.4)}$$

> $eq[35\,a] := P[-a,c]\cdot P[-b,d]\cdot dotomega[-c,-d] = dotomega[-a,-b] : T(\%);$

$$P_a{}^c\, P_b{}^d\, dotomega_{c\,d} = dotomega_{a\,b} \quad \textbf{(1.5)}$$

> $eq[35\,b] := dotomega[-a,-b] = -\left(\frac{2}{3}\right)\cdot\mathrm{theta}\cdot\mathrm{omega}[-a,-b] : T(\%);$

$$dotomega_{a\,b} = -\frac{2}{3}\,\theta\,\omega_{a\,b} \quad \textbf{(1.6)}$$

> $eq[36\,a] := P[a,-b]\cdot dotomega[b] = dotomega[a] : T(\%);$

$$P^a{}_b\, dotomega^b = dotomega^a \quad \textbf{(1.7)}$$

> $eq[36\,b] := dotomega[a] = -\left(\frac{2}{3}\right)\cdot\mathrm{theta}\cdot\mathrm{omega}[a] : T(\%);$

$$dotomega^a = -\frac{2}{3}\,\theta\,\omega^a \quad \textbf{(1.8)}$$

> $eq[36\,c] := dotomega = -\left(\frac{2}{3}\right)\cdot\mathrm{theta}\cdot\mathrm{omega} : T(\%);$

$$dotomega = -\frac{2}{3}\,\theta\,\omega \quad \textbf{(1.9)}$$

> $eq[36] := dotomega = -\left(\frac{2}{3}\right)\cdot\mathrm{theta}\cdot\mathrm{omega} : T(\%);$

$$dotomega = -\frac{2}{3}\,\theta\,\omega \quad \textbf{(1.10)}$$

> $eq[37] := \mathrm{theta}\cdot\left(c1p - \frac{c2}{3}\cdot\mathrm{omega}\cdot\mathrm{omega}\right) = 0 : T(\%);$

$$\theta\left(c1p - \frac{1}{3}\,c2\,\omega^2\right) = 0 \quad \textbf{(1.11)}$$

> **save** *eq*, "Seneqs2e2" :

go to page 3

```
> read "Seneqs2e2";
```

$$eq := table\Big(\Big[1 = \big(TensorPack{:-}T_{-a,\,-b} = \rho\, u_{-a}\, u_{-b}\big), 2 = \big(P_{-a,\,-b} = u_{-a}\, u_{-b} + g_{-a,\,-b}\big), 3 = \big(P_{a,\,-b}\, u_{b} = 0\big), 4 = \big(dX_{a} = u_{b}\, X_{a,\,-B}\big), 5 = \big(du_{a} = u_{b}\, u_{a,\,-B}\big), 10\,a = \Big(\omega_{b} = \frac{1}{2}\,\eta_{b,\,e,\,f,\,g}\, u_{-e}\, \omega_{-f,\,-g}\Big), 6 = \Big(u_{-a,\,-B} = \frac{1}{3}\,\theta\, P_{-a,\,-b} + \sigma_{-a,\,-b} + \omega_{-a,\,-b} - du_{-a}\, u_{-b}\Big), 7 = \big(\theta = u_{a,\,-A}\big), 9 = \Big(\omega_{-a,\,-b} = \frac{1}{2}\,P_{-a,\,c}\, P_{-b,\,d}\, u_{-c,\,-D} - \frac{1}{2}\,P_{-b,\,c}\, P_{-a,\,d}\, u_{-c,\,-D}\Big), 8 = \Big(\sigma_{-a,\,-b} = \frac{1}{2}\,P_{-a,\,c}\, P_{-b,\,d}\, u_{-c,\,-D} + \frac{1}{2}\,P_{-b,\,c}\, P_{-a,\,d}\, u_{-c,\,-D} - \frac{1}{3}\,\theta\, P_{-a,\,-b}\Big), 11 = \big(\omega_{-a,\,-b} = \eta_{-a,\,-b,\,-e,\,-f}\,\omega_{e}\, u_{f}\big), 12\,a = \Big(\omega^{2} = \frac{1}{2}\,\omega_{a,\,b}\,\omega_{-a,\,-b}\Big), 10 = \Big(\omega_{a} = \frac{1}{2}\,\eta_{a,\,b,\,c,\,d}\, u_{-b}\,\omega_{-c,\,-d}\Big), 13 = \text{"iff(iff(omega[-a,-b] = 0,omega[-a]),omega = 0)"}, 12 = \Big(\omega^{2} = \frac{1}{2}\,\omega_{a,\,b}\,\omega_{-a,\,-b}\Big), 15 = \Big(\frac{1}{2}\,u_{-b,\,-A} - \frac{1}{2}\,u_{-a,\,-B} = \frac{1}{2}\,du_{-a}\, u_{-b} - \frac{1}{2}\,du_{-b}\, u_{-a} + \omega_{a,\,b}\Big), 14 = \big(\omega_{-a,\,c}\,\omega_{-c,\,b} = -\omega^{2}\, P_{-a,\,b} + \omega_{b}\,\omega_{-a}\big), 18 = \Big(u_{-b,\,-A} = -u_{-a}\, u_{-b,\,-C}\, u_{c} + \frac{1}{3}\,\theta\, h_{-a,\,-b} + \omega_{-a,\,-b}\Big), 19 = \big(u_{a,\,-C,\,-D} - u_{a,\,-D,\,-C} = R_{a,\,-b,\,-c,\,-d}\, u_{b}\big), 16 = \Big(-\frac{1}{6}\,u_{-c}\, u_{-a,\,-B} + \frac{1}{6}\,u_{-c}\, u_{-b,\,-A} + \frac{1}{6}\,u_{-b}\, u_{-a,\,-C} - \frac{1}{6}\,u_{-b}\, u_{-c,\,-A} - \frac{1}{6}\,u_{-a}\, u_{-b,\,-C} + \frac{1}{6}\,u_{-a}\, u_{-c,\,-B} = 0\Big), 17 = \big(\sigma_{-a,\,-b} = 0\big), 22 = \Big(\omega_{-a}\,\omega_{-b} - \frac{1}{3}\,P_{-a,\,-b}\,\omega^{2} + E_{-a,\,-b} = 0\Big), 23 = \big(E_{-a,\,-b} = C_{-a,\,-b,\,-c,\,-d}\, u_{c}\, u_{d}\big), 20 = \Big(dottheta + \frac{1}{3}\,\theta^{2} - 2\,\omega^{2} + \frac{1}{2}\,\mu = 0\Big), 21 = \Big(P_{-a,\,c}\, P_{-b,\,d}\,\omega_{-c,\,-d,\,-F}\, u_{f} + \frac{2}{3}\,\theta\,\omega_{-a,\,-b} = 0\Big), 27 = \big(\omega_{a,\,-A} = 2\, du_{a}\,\omega_{-a}\big), 26 = 2\, P_{a,\,b}\,\theta_{-B} + 3\, P_{a,\,-b}\,\omega_{b,\,d,\,-D}, 25 = \Big(P_{a,\,-b}\,\omega_{b,\,-F}\, u_{f} + \frac{2}{3}\,\theta\,\omega_{a} = 0\Big), 24 = \Big(H_{-a,\,-b} = \frac{1}{2}\,\eta_{-a,\,-e,\,c,\,d}\, C_{-c,\,-d,\,-b,\,-f}\, u_{e}\, u_{f}\Big), 31 = (\mu + p)\, du_{a} + P_{a,\,b}\, p_{-B}, 30 = \big(\mu\,\theta + dotmu = 0\big), 29 = \big(\omega_{-a,\,-n}\,\omega_{n,\,m,\,-M} = \omega^{2}\, du_{-a} + P_{-a,\,b}\,\omega_{c}\,\omega_{-b,\,-C} - P_{-a,\,b}\,\omega_{c}\,\omega_{-c,\,-B} \qquad \textbf{(1.12)}$$

$- du_{-p}\, \omega_p\, \omega_{-a}\big)$, $36\ c = \left(dotomega = -\frac{2}{3}\ \theta\ \omega\right)$, $7\ a = \left(\theta = u_{d,\ -D}\right)$, $28 = \Big(H_{-a,\ -b}$

$= \frac{1}{2}\ P_{-a,\ c} P_{-b,\ d}\, \omega_{d,\ C} + \frac{1}{2}\ P_{-b,\ c} P_{-a,\ d}\, \omega_{d,\ C}\Big)$, $36 = \left(dotomega = -\frac{2}{3}\ \theta\ \omega\right)$, 37

$= \left(\theta\ \left(c1p - \frac{1}{3}\ c2\ \omega^2\right) = 0\right)$, $32 = \left(du_a = 0\right)$, $33 = \left(u_{-a} = -\frac{f_{-A}}{fdot}\right)$, $34 = \big(\mu = (c1$

$-1)\ p + c2\ \omega^2\big)$, $35 = \left(dotomega_{-a,\ -b} = -\frac{2}{3}\ \theta\ \omega_{-a,\ -b}\right)$, $27\ a = \left(\omega_{a,\ -A} = 0\right)$, $14\ a$

$= \left(\omega_{-a,\ c}\, \omega_{-c,\ b} = -\omega^2\ P_{-a,\ b} + \omega_b\, \omega_{-a}\right)$, $11\ m = \left(\omega_{-a,\ -b} = \eta_{-a,\ -b,\ -c,\ -d}\, u_d\, \omega_c\right)$, $10\ b$

$= \left(\eta_{-f,\ -g,\ -a,\ -e}\, \omega_a\, u_e = \frac{1}{2}\ \eta_{-f,\ -g,\ -a,\ -e}\, \eta_{a,\ b,\ c,\ d}\, u_{-b}\, \omega_{-c,\ -d}\, u_e\right)$, $35\ b = \Big(dotomega_{-a,\ -b}$

$= -\frac{2}{3}\ \theta\ \omega_{-a,\ -b}\Big)$, $11\ m2 = \left(\omega_{a,\ b} = \eta_{a,\ b,\ -e,\ -f}\, u_f\, \omega_e\right)$, $16\ a = \Big(-\frac{1}{6}\ u_{-c}\, u_{-a,\ -B}$

$+ \frac{1}{6}\ u_{-c}\, u_{-b,\ -A} + \frac{1}{6}\ u_{-b}\, u_{-a,\ -C} - \frac{1}{6}\ u_{-b}\, u_{-c,\ -A} - \frac{1}{6}\ u_{-a}\, u_{-b,\ -C} + \frac{1}{6}\ u_{-a}\, u_{-c,\ -B}$

$= 0\Big)$, $27\ b = \left(\eta_{a,\ b,\ c,\ d}\, u_{-a}\, \omega_{-c,\ -d,\ -B} = 0\right)$, $14\ b = \left(\omega_{-a,\ c}\, \omega_{-c,\ d}\, \omega_{-d,\ b} = -\omega^2\ \omega_{-a,\ b}\right)$,

$35\ a = \left(P_{-a,\ c} P_{-b,\ d}\ dotomega_{-c,\ -d} = dotomega_{-a,\ -b}\right)$, $11\ m1 = \Big(\omega_{a,\ -b}$

$= \eta_{a,\ -b,\ -c,\ -d}\, u_d\, \omega_c\Big)$, $12\ b = \left(\omega^2 = \omega_a\, \omega_{-a}\right)$, $36\ a = \left(P_{a,\ -b}\ dotomega_b = dotomega_a\right)$,

$36\ b = \left(dotomega_a = -\frac{2}{3}\ \theta\ \omega_a\right)$, $16\ b = \left(\omega_{-a,\ -b} = 0\right)\big]\big)$

> PrintSubArray(eq, 1, 37, y);

$$1,\ T_{a\,b} = \rho\, u_a\, u_b$$

$$2,\ P_{a\,b} = u\, u_{b\,a} + g_{b\,a}$$

$$3,\ P^a{}_b\, u^b = 0$$

$$4,\ dX^a = u^b\, X^a{}_{;b}$$

$$5,\ du^a = u^b\, u^a{}_{;b}$$

$$6,\ u_{a\,;b} = \frac{1}{3}\ \theta\, P_{a\,b} + \sigma_{a\,b} + \omega_{a\,b} - du_a\, u_b$$

$$7,\ \theta = u^a{}_{;a}$$

$$8,\ \sigma_{a\,b} = \frac{1}{2}\ P_a{}^c P_b{}^d\, u_{c\,;d} + \frac{1}{2}\ P_b{}^c P_a{}^d\, u_{c\,;d} - \frac{1}{3}\ \theta\, P_{a\,b}$$

$$9,\ \omega_{ab} = \frac{1}{2} P_a{}^c P_b{}^d u_{c;d} - \frac{1}{2} P_b{}^c P_a{}^d u_{c;d}$$

$$10,\ \omega^a = \frac{1}{2} \eta^{abcd} u_b \omega_{cd}$$

$$11,\ \omega_{ab} = \eta_{abef} \omega^e u^f$$

$$12,\ \omega^2 = \frac{1}{2} \omega^{ab} \omega_{ab}$$

13, "iff(iff(omega[-a,-b] = 0,omega[-a]),omega = 0)"

$$14,\ \omega_a{}^c \omega_c{}^b = -\omega^2 P_a{}^b + \omega^b \omega_a$$

$$15,\ \frac{1}{2} u_{b;a} - \frac{1}{2} u_{a;b} = \frac{1}{2} du_a u_b - \frac{1}{2} du_b u_a + \omega^{ab}$$

$$16,\ -\frac{1}{6} u_c u_{a;b} + \frac{1}{6} u_c u_{b;a} + \frac{1}{6} u_b u_{a;c} - \frac{1}{6} u_b u_{c;a} - \frac{1}{6} u_a u_{b;c} + \frac{1}{6} u_a u_{c;b} = 0$$

$$17,\ \sigma_{ab} = 0$$

$$18,\ u_{b;a} = -u_a u_{b;c} u^c + \frac{1}{3} \theta h_{ab} + \omega_{ab}$$

$$19,\ u^a{}_{;c;d} - u^a{}_{;d;c} = R^a{}_{bcd} u^b$$

$$20,\ dottheta + \frac{1}{3} \theta^2 - 2\omega^2 + \frac{1}{2} \mu = 0$$

$$21,\ P_a{}^c P_b{}^d \omega_{cd;f} u^f + \frac{2}{3} \theta \omega_{ab} = 0$$

$$22,\ \omega_a \omega_b - \frac{1}{3} P_{ab} \omega^2 + E_{ab} = 0$$

$$23,\ E_{ab} = C_{abcd} u^c u^d$$

$$24,\ H_{ab} = \frac{1}{2} \eta_{ae}{}^{cd} C_{cdbf} u^e u^f$$

$$25,\ P^a{}_b \omega^b{}_{;f} u^f + \frac{2}{3} \theta \omega^a = 0$$

$$26,\ 2 P^{ab} \theta_{;b} + 3 P^a{}_b \omega^{bd}{}_{;d}$$

$$27,\ \omega^a{}_{;a} = 2\, du^a \omega_a$$

$$28,\ H_{ab} = \frac{1}{2} P_a{}^c P_b{}^d \omega^{d\ ;c} + \frac{1}{2} P_b{}^c P_a{}^d \omega^{d\ ;c}$$

$$29,\ \omega_{an} \omega^{nm}{}_{;m} = \omega^2 du_a + P_a{}^b \omega^c \omega_{b;c} - P_a{}^b \omega^c \omega_{c;b} - du_p \omega^p \omega_a$$

$$30,\ \mu\theta + dotmu = 0$$

$$31,\ (\mu + p)\, du^a + P^{ab} p_{;b}$$

$$32, du^{a} = 0$$

$$33, u_{a} = -\frac{f_{;a}}{fdot}$$

$$34, \mu = (c1 - 1)\, p + c2\, \omega^{2}$$

$$35, dotomega_{a\,b} = -\frac{2}{3}\, \theta\, \omega_{a\,b}$$

$$36, dotomega = -\frac{2}{3}\, \theta\, \omega$$

$$37, \theta \left(c1p - \frac{1}{3}\, c2\, \omega^{2} \right) = 0 \qquad \textbf{(1.13)}$$

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