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> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index) :

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Chapter XX

Tensor analysis using indices - Senovilla et al. - Shearfree for dust

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if $\sigma_{ab} = 0 \Rightarrow \omega \Theta = 0$

Author: Peter Huf

file 2e:SSSeqs30-31

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> read "EFE" : read "SFE" : read "fids" : read "Seneqs2e" :

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Equation 30 - from literature

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> eq[30] := mu[F]·u[-f] + mu·theta = 0 : T(%);

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$$\theta \mu + \mu^i u_f = 0 \quad (1.1)$$

OR

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> eq[30] := dotmu + mu·theta = 0 : T(%);

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$$\mu \theta + \text{dotmu} = 0 \quad (1.2)$$

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Equation 31

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eq31 is an Einstein field equation and is common in the main literature (Ellis (1970))

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> eq[31] := (mu + p)·du[a] + P[a, b]·p[-B] : T(%);

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$$(\mu + p) du^a + P^a_b p^b = 0 \quad (1.3)$$

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> save eq, "Seneqs2e1" :

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go to page 3

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> read "Seneqs2e1";

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eq := table( [ 1 = (TensorPack:-T_{-a, -b} = \rho u_{-a} u_{-b}), 2 = (P_{-a, -b} = u_{-a} u_{-b} + g_{-a, -b}), 3

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$$= (P_{a, -b} u_b = 0), 4 = (\omega_{-a, c} \omega_{-c, d} \omega_{-d, b} = -\omega^2_{-a, b}), 5 = (dX_a = u_b X_{a, -B}), 5$$

$$\begin{aligned}
&= (du_a = u_b u_{a, -B}), 6 = \left(u_{-a, -B} = \frac{1}{3} \theta P_{-a, -b} + \sigma_{-a, -b} + \omega_{-a, -b} - du_{-a} u_{-b} \right), 7 = (\theta \\
&= u_{a, -A}), 9 = \left(\omega_{-a, -b} = \frac{1}{2} P_{-a, c} P_{-b, d} u_{-c, -D} - \frac{1}{2} P_{-b, c} P_{-a, d} u_{-c, -D} \right), 8 = \left(\sigma_{-a, -b} \right. \\
&= \frac{1}{2} P_{-a, c} P_{-b, d} u_{-c, -D} + \frac{1}{2} P_{-b, c} P_{-a, d} u_{-c, -D} - \frac{1}{3} \theta P_{-a, -b} \left. \right), 11 = (\omega_{-a, -b} \\
&= \eta_{-a, -b, -e, -f} \omega_e u_f), 10 = \left(\omega_a = \frac{1}{2} \eta_{a, b, c, d} u_{-b} \omega_{-c, -d} \right), 13 \\
&= \text{"iff(iff(omega[-a,-b] = 0, omega[-a]), omega = 0)"}, 12 = \left(\omega^2 = \frac{1}{2} \omega_{a, b} \omega_{-a, -b} \right), 15 \\
&= \left(\frac{1}{2} u_{-b, -A} - \frac{1}{2} u_{-a, -B} = \frac{1}{2} du_{-a} u_{-b} - \frac{1}{2} du_{-b} u_{-a} + \omega_{a, b} \right), 14 = \left(\omega_{-a, c} \omega_{-c, b} = \right. \\
&- \omega^2 P_{-a, b} + \omega_b \omega_{-a} \left. \right), 11 m = (\omega_{-a, -b} = \eta_{-a, -b, -c, -d} u_d \omega_c), 18 = \left(u_{-b, -A} = \right. \\
&- u_{-a} u_{-b, -C} u_c + \frac{1}{3} \theta h_{-a, -b} + \omega_{-a, -b} \left. \right), 12 b = (\omega^2 = \omega_a \omega_{-a}), 19 = (u_{a, -C, -D} \\
&- u_{a, -D, -C} = R_{a, -b, -c, -d} u_b), 16 = \left(-\frac{1}{6} u_{-c} u_{-a, -B} + \frac{1}{6} u_{-c} u_{-b, -A} + \frac{1}{6} u_{-b} u_{-a, -C} \right. \\
&- \frac{1}{6} u_{-b} u_{-c, -A} - \frac{1}{6} u_{-a} u_{-b, -C} + \frac{1}{6} u_{-a} u_{-c, -B} = 0 \left. \right), 17 = (\sigma_{-a, -b} = 0), 22 \\
&= \left(\omega_{-a} \omega_{-b} - \frac{1}{3} P_{-a, -b} \omega^2 + E_{-a, -b} = 0 \right), 23 = (E_{-a, -b} = C_{-a, -b, -c, -d} u_c u_d), 20 \\
&= \left(\text{dottheta} + \frac{1}{3} \theta^2 - 2 \omega^2 + \frac{1}{2} \mu = 0 \right), 21 = \left(P_{-a, c} P_{-b, d} \omega_{-c, -d, -F} u_f + \frac{2}{3} \theta \omega_{-a, -b} \right. \\
&= 0 \left. \right), 27 = (\omega_{a, -A} = 2 du_a \omega_{-a}), 26 = 2 P_{a, b} \theta_{-B} + 3 P_{a, -b} \omega_{b, d, -D}, 25 \\
&= \left(P_{a, -b} \omega_{b, -F} u_f + \frac{2}{3} \theta \omega_a = 0 \right), 24 = \left(H_{-a, -b} = \frac{1}{2} \eta_{-a, -e, c, d} C_{-c, -d, -b, -f} u_e u_f \right), 31 \\
&= (\mu + p) du_a + P_{a, b} p_{-B}, 30 = (\mu \theta + \text{dotmu} = 0), 29 = \left(\omega_{-a, -n} \omega_{n, m, -M} = \omega^2 du_{-a} \right. \\
&+ P_{-a, b} \omega_c \omega_{-b, -C} - P_{-a, b} \omega_c \omega_{-c, -B} - du_{-p} \omega_p \omega_{-a} \left. \right), 28 = \left(H_{-a, -b} \right. \\
&= \frac{1}{2} P_{-a, c} P_{-b, d} \omega_{d, C} + \frac{1}{2} P_{-b, c} P_{-a, d} \omega_{d, C} \left. \right), 27 a = (\omega_{a, -A} = 0), 16 a = \left(\right. \\
&- \frac{1}{6} u_{-c} u_{-a, -B} + \frac{1}{6} u_{-c} u_{-b, -A} + \frac{1}{6} u_{-b} u_{-a, -C} - \frac{1}{6} u_{-b} u_{-c, -A} - \frac{1}{6} u_{-a} u_{-b, -C} \\
&+ \frac{1}{6} u_{-a} u_{-c, -B} = 0 \left. \right), 16 b = (\omega_{-a, -b} = 0), 11 m2 = (\omega_{a, b} = \eta_{a, b, -e, -f} u_f \omega_e), 14 a \\
&= \left(\omega_{-a, c} \omega_{-c, b} = -\omega^2 P_{-a, b} + \omega_b \omega_{-a} \right), 7 a = (\theta = u_{d, -D}), 10 a = (\omega_b
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \eta_{b, e, f, g} u_{-e} \omega_{-f, -g}, 27 b = (\eta_{a, b, c, d} u_{-a} \omega_{-c, -d, -B} = 0), 12 a = (\omega^2 \\
&= \frac{1}{2} \omega_{a, b} \omega_{-a, -b}), 10 b = (\eta_{-f, -g, -a, -e} \omega_a u_e \\
&= \frac{1}{2} \eta_{-f, -g, -a, -e} \eta_{a, b, c, d} u_{-b} \omega_{-c, -d} u_e), 11 m l = (\omega_{a, -b} = \eta_{a, -b, -c, -d} u_d \omega_c) \Big]
\end{aligned}$$

> PrintSubArray(eq, 1, 31, y);

$$1, T_{ab} = \rho u_a u_b$$

$$2, P_{ab} = u u_{ab} + g_{ab}$$

$$3, P^a_b u^b = 0$$

$$4, dX^a = u^b X^a_{;b}$$

$$5, du^a = u^b u^a_{;b}$$

$$6, u_{a;b} = \frac{1}{3} \theta P_{ab} + \sigma_{ab} + \omega_{ab} - du_a u_b$$

$$7, \theta = u^a_{;a}$$

$$8, \sigma_{ab} = \frac{1}{2} P_a^c P_b^d u_{c;d} + \frac{1}{2} P_b^c P_a^d u_{c;d} - \frac{1}{3} \theta P_{ab}$$

$$9, \omega_{ab} = \frac{1}{2} P_a^c P_b^d u_{c;d} - \frac{1}{2} P_b^c P_a^d u_{c;d}$$

$$10, \omega^a = \frac{1}{2} \eta^{abcd} u_b \omega_{cd}$$

$$11, \omega_{ab} = \eta_{abef} \omega^e u^f$$

$$12, \omega^2 = \frac{1}{2} \omega^a_b \omega_{ab}$$

$$13, \text{"iff(iff(omega[-a,-b] = 0, omega[-a]), omega = 0)"}$$

$$14, \omega_a^c \omega_c^b = -\omega^2 P_a^b + \omega^b \omega_a$$

$$15, \frac{1}{2} u_{b;a} - \frac{1}{2} u_{a;b} = \frac{1}{2} du_a u_b - \frac{1}{2} du_b u_a + \omega^a_b$$

$$16, -\frac{1}{6} u_c u_{a;b} + \frac{1}{6} u_c u_{b;a} + \frac{1}{6} u_b u_{a;c} - \frac{1}{6} u_b u_{c;a} - \frac{1}{6} u_a u_{b;c} + \frac{1}{6} u_a u_{c;b} = 0$$

$$17, \sigma_{ab} = 0$$

$$18, u_{b;a} = -u_a u_{b;c} u^c + \frac{1}{3} \theta h_{ab} + \omega_{ab}$$

$$19, u^a_{;c;d} - u^a_{;d;c} = R^a_{bcd} u^b$$

$$20, \text{dottheta} + \frac{1}{3} \theta^2 - 2 \omega^2 + \frac{1}{2} \mu = 0$$

$$21, P_a{}^c P_b{}^d \omega_{cd;f} u^f + \frac{2}{3} \theta \omega_{ab} = 0$$

$$22, \omega_a \omega_b - \frac{1}{3} P_{ab} \omega^2 + E_{ab} = 0$$

$$23, E_{ab} = C_{abcd} u^c u^d$$

$$24, H_{ab} = \frac{1}{2} \eta_{ae}{}^{cd} C_{cdbf} u^e u^f$$

$$25, P_a{}^b \omega_{;f} u^f + \frac{2}{3} \theta \omega^a = 0$$

$$26, 2 P_a{}^b \theta_{;b} + 3 P_a{}^b \omega^b{}_{;d}$$

$$27, \omega^a{}_{;a} = 2 du^a \omega_a$$

$$28, H_{ab} = \frac{1}{2} P_a{}^c P_b{}^d \omega^d{}_{;c} + \frac{1}{2} P_b{}^c P_a{}^d \omega^d{}_{;c}$$

$$29, \omega_{an} \omega^{nm}{}_{;m} = \omega^2 du_a + P_a{}^b \omega^c \omega_{b;c} - P_a{}^b \omega^c \omega_{c;b} - du_p \omega^p \omega_a$$

$$30, \mu \theta + \text{dotmu} = 0$$

$$31, (\mu + p) du^a + P_a{}^b p_{;b}$$

(1.5)

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