```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index) :
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for dust
page 2

if $\sigma_{a\,b}$=0 =>ωΘ=0
Author: Peter Huf
file 2d:eqs 26-30

In this file we continue to follow the equations outlined by Senovilla et al. (2007) with the assumptions for dust
i.e

```
> read "EFE" : read "SFE" :read "fids" :read "Seneqs2c" :
>
```

****
Equation 26
****

The constraint equations are (with dust assumptions) :

```
> eq[26] := 2· P[a, b]·theta[-B] + 3·P[a,-b]·omega[b, d,-D] = 0 : T(%);
```

$$2\,P^{a\ b}\,\theta_{;b} + 3\,P^{a}{}_{b}\,\omega^{b\ d}{}_{;d} = 0 \qquad (1.1)$$

Proof of eq26:

This equation is derived as eq69 of Ellis 1970 (where shear and acceleration are zero):

```
> Eeq[69] := (2/3)· P[a,-b]·theta[B] + -eta[a, b, d, f]·u[-b]·omega[-d,-F] = 0 : T(%);
```

$$\frac{2}{3}\,P^{a}{}_{b}\,\theta^{;b} - \eta^{a\ b\ d\ f}\,u_{b}\,\omega_{d;f} = 0 \qquad (1.2)$$

Now we subsitute the identity:

> temp1 := omega[-d] = 1/2 · eta[-d, g, h, i]·u[-g]·omega[-h,-i] : T(%);

$$\omega_{d} = \frac{1}{2}\eta_{d}{}^{g\,h\,i}u_{g}\omega_{h\,i} \tag{1.3}$$

> temp2 := cod(temp1,-f) : T(%);

$$\omega_{d;f} = \frac{1}{2}\eta_{d}{}^{g\,h\,i}{}_{;f}u_{g}\omega_{h\,i} + \frac{1}{2}\eta_{d}{}^{g\,h\,i}u_{g;f}\omega_{h\,i} + \frac{1}{2}\eta_{d}{}^{g\,h\,i}u_{g}\omega_{h\,i;f} \tag{1.4}$$

> temp3 := expand(TEDS(eta[-d, g, h, i,-F] = 0, temp2)) : T(%);

$$\omega_{d;f} = \frac{1}{2}\eta_{d}{}^{g\,h\,i}u_{g;f}\omega_{h\,i} + \frac{1}{2}\eta_{d}{}^{g\,h\,i}u_{g}\omega_{h\,i;f} \tag{1.5}$$

> temp4 := expand(TEDS(temp3, Eeq[69])) : T(%);

$$\frac{2}{3}P^{a}{}_{b}\theta^{;b} - \frac{1}{2}\eta^{a\,b\,d\,f}u_{b}\eta_{d}{}^{g\,h\,i}u_{g;f}\omega_{h\,i} - \frac{1}{2}\eta^{a\,b\,d\,f}u_{b}\eta_{d}{}^{g\,h\,i}u_{g}\omega_{h\,i;f} = 0 \tag{1.6}$$

(swapping one pair of indices:)

> temp5 := eta[a, b, d, f]·eta[-d, g, h, i] =-1· g[g, k]·g[h, l]·g[i, m]·eta[d, b, a, f]·eta[-d,-k, -l,-m] : T(%);

$$\eta^{a\,b\,d\,f}\eta_{d}{}^{g\,h\,i} = -g^{g\,k}g^{h\,l}g^{i\,m}\eta^{d\,b\,a\,f}\eta_{d\,k\,l\,m} \tag{1.7}$$

> temp6 := eta[d, b, a, f]·eta[-d,-k,-l,-m] =-6·antisymm(delta[b,-k]·delta[a,-l]·delta[f, -m], b, f) : T(%);

$$\eta^{d\,b\,a\,f}\eta_{d\,k\,l\,m} = \delta^{a}{}_{k}\delta^{b}{}_{l}\delta^{f}{}_{m} - \delta^{a}{}_{k}\delta^{f}{}_{l}\delta^{b}{}_{m} - \delta^{b}{}_{k}\delta^{a}{}_{l}\delta^{f}{}_{m} + \delta^{f}{}_{k}\delta^{a}{}_{l}\delta^{b}{}_{m} + \delta^{b}{}_{k}\delta^{f}{}_{l}\delta^{a}{}_{m} - \delta^{f}{}_{k}\delta^{b}{}_{l}\delta^{a}{}_{m} \tag{1.8}$$

> temp7 := expand(TEDS(temp6, temp5)) : T(%);

$$\eta^{a\,b\,d\,f}\eta_{d}{}^{g\,h\,i} = -\delta^{a}{}_{k}\delta^{b}{}_{l}\delta^{f}{}_{m}g^{g\,k}g^{h\,l}g^{i\,m} + \delta^{a}{}_{k}\delta^{b}{}_{m}\delta^{f}{}_{l}g^{g\,k}g^{h\,l}g^{i\,m} + \delta^{a}{}_{l}\delta^{b}{}_{k}\delta^{f}{}_{m}g^{g\,k}g^{h\,l}g^{i\,m} - \delta^{a}{}_{l}\delta^{b}{}_{m}\delta^{f}{}_{k}g^{g\,k}g^{h\,l}g^{i\,m} - \delta^{a}{}_{m}\delta^{b}{}_{k}\delta^{f}{}_{l}g^{g\,k}g^{h\,l}g^{i\,m} + \delta^{a}{}_{m}\delta^{b}{}_{l}\delta^{f}{}_{k}g^{g\,k}g^{h\,l}g^{i\,m} \tag{1.9}$$

> proof[26 a] := expand(TEDS(temp7, temp4)) : T(%);

$$\frac{2}{3}P^{a}{}_{b}\theta^{;b} + \frac{1}{2}\delta^{a}{}_{k}\delta^{b}{}_{l}\delta^{f}{}_{m}g^{g\,k}g^{h\,l}g^{i\,m}\omega_{h\,i}u_{b}u_{g;f} - \frac{1}{2}\delta^{a}{}_{k}\delta^{b}{}_{m}\delta^{f}{}_{l}g^{g\,k}g^{h\,l}g^{i\,m}\omega_{h\,i}u_{b}u_{g;f} - \frac{1}{2}\delta^{a}{}_{l}\delta^{b}{}_{k}\delta^{f}{}_{m}g^{g\,k}g^{h\,l}g^{i\,m}\omega_{h\,i}u_{b}u_{g;f} + \frac{1}{2}\delta^{a}{}_{l}\delta^{b}{}_{m}\delta^{f}{}_{k}g^{g\,k}g^{h\,l}g^{i\,m}\omega_{h\,i}u_{b}u_{g;f} + \frac{1}{2}\delta^{a}{}_{m}\delta^{b}{}_{k}\delta^{f}{}_{l}g^{g\,k}g^{h\,l}g^{i\,m}\omega_{h\,i}u_{b}u_{g;f} \tag{1.10}$$

$$-\frac{1}{2}\,\delta^{a}{}_{m}\,\delta^{b}{}_{l}\,\delta^{f}{}_{k}\,g^{g\,k}\,g^{h\,l}\,g^{i\,m}\,\omega_{h\,i}\,u_{b}\,u_{g\,;f}$$

$$+\frac{1}{2}\,\delta^{a}{}_{k}\,\delta^{b}{}_{l}\,\delta^{f}{}_{m}\,g^{g\,k}\,g^{h\,l}\,g^{i\,m}\,\omega_{h\,i\,;f}\,u_{b}\,u_{g}$$

$$-\frac{1}{2}\,\delta^{a}{}_{k}\,\delta^{b}{}_{m}\,\delta^{f}{}_{l}\,g^{g\,k}\,g^{h\,l}\,g^{i\,m}\,\omega_{h\,i\,;f}\,u_{b}\,u_{g}$$

$$-\frac{1}{2}\,\delta^{a}{}_{l}\,\delta^{b}{}_{k}\,\delta^{f}{}_{m}\,g^{g\,k}\,g^{h\,l}\,g^{i\,m}\,\omega_{h\,i\,;f}\,u_{b}\,u_{g}$$

$$+\frac{1}{2}\,\delta^{a}{}_{l}\,\delta^{b}{}_{m}\,\delta^{f}{}_{k}\,g^{g\,k}\,g^{h\,l}\,g^{i\,m}\,\omega_{h\,i\,;f}\,u_{b}\,u_{g}$$

$$+\frac{1}{2}\,\delta^{a}{}_{m}\,\delta^{b}{}_{k}\,\delta^{f}{}_{l}\,g^{g\,k}\,g^{h\,l}\,g^{i\,m}\,\omega_{h\,i\,;f}\,u_{b}\,u_{g}$$

$$-\frac{1}{2}\,\delta^{a}{}_{m}\,\delta^{b}{}_{l}\,\delta^{f}{}_{k}\,g^{g\,k}\,g^{h\,l}\,g^{i\,m}\,\omega_{h\,i\,;f}\,u_{b}\,u_{g}=0$$

```
> proof[26 b]
    := Absorbg(Absorbg(Absorbg(Absorbd(Absorbd(Absorbd(proof[26 a])))))) : T(%);
```

0, "not a tensor"

0, "not a tensor"

0, "not a tensor"

$$\frac{2}{3}\,P^{a}{}_{b}\,\theta^{;b}+\frac{1}{2}\,\omega^{l\,m}\,u_{l}\,u^{a}{}_{;m}-\frac{1}{2}\,\omega^{l\,m}\,u_{m}\,u^{a}{}_{;l}-\frac{1}{2}\,\omega^{a\,m}\,u_{k}\,u^{k}{}_{;m}$$
$$+\frac{1}{2}\,\omega^{a\,m}\,u_{m}\,u^{k}{}_{;k}+\frac{1}{2}\,\omega^{l\,a}\,u_{k}\,u^{k}{}_{;l}-\frac{1}{2}\,\omega^{l\,a}\,u_{l}\,u^{k}{}_{;k}+\frac{1}{2}\,\omega^{l\,m}{}_{;m}\,u_{l}\,u^{a}$$
$$-\frac{1}{2}\,\omega^{l\,m}{}_{;l}\,u_{m}\,u^{a}-\frac{1}{2}\,\omega^{a\,m}{}_{;m}\,u_{k}\,u^{k}+\frac{1}{2}\,\omega^{a\,m}{}_{;k}\,u_{m}\,u^{k}+\frac{1}{2}\,\omega^{l\,a}{}_{;l}\,u_{k}\,u^{k}$$
$$-\frac{1}{2}\,\omega^{l\,a}{}_{;k}\,u_{l}\,u^{k}=0 \tag{1.11}$$

```
> proof[26 c] := expand(TEDS(u[-k]·u[k]=-1, proof[26 b])) : T(%);
```

$$\frac{1}{2}\,\omega^{a\,m}{}_{;k}\,u_{m}\,u^{k}-\frac{1}{2}\,\omega^{l\,a}{}_{;k}\,u_{l}\,u^{k}+\frac{2}{3}\,P^{a}{}_{b}\,\theta^{;b}+\frac{1}{2}\,\omega^{l\,m}\,u_{l}\,u^{a}{}_{;m}$$
$$-\frac{1}{2}\,\omega^{l\,m}\,u_{m}\,u^{a}{}_{;l}+\frac{1}{2}\,\omega^{a\,m}\,u_{m}\,u^{k}{}_{;k}-\frac{1}{2}\,\omega^{l\,a}\,u_{l}\,u^{k}{}_{;k}+\frac{1}{2}\,\omega^{l\,m}{}_{;m}\,u_{l}\,u^{a}$$
$$-\frac{1}{2}\,\omega^{l\,m}{}_{;l}\,u_{m}\,u^{a}+\frac{1}{2}\,\omega^{a\,m}{}_{;m}-\frac{1}{2}\,\omega^{l\,a}{}_{;l}-\frac{1}{2}\,\omega^{a\,m}\,u_{k}\,u^{k}{}_{;m}$$
$$+\frac{1}{2}\,\omega^{l\,a}\,u_{k}\,u^{k}{}_{;l}=0 \tag{1.12}$$

```
> proof[26 d] := expand(TEDS(u[-l]·omega[l, m]=0, proof[26 c])) : T(%);
```

$$\frac{1}{2}\,\omega^{a\,m}{}_{;k}\,u_{m}\,u^{k}-\frac{1}{2}\,\omega^{l\,a}{}_{;k}\,u_{l}\,u^{k}+\frac{2}{3}\,P^{a}{}_{b}\,\theta^{;b}-\frac{1}{2}\,\omega^{l\,m}\,u_{m}\,u^{a}{}_{;l} \tag{1.13}$$

$$+\frac{1}{2}\,\omega^{a\;m}\,u_m\,u^k{}_{;k}-\frac{1}{2}\,\omega^{l\;a}\,u_l\,u^k{}_{;k}+\frac{1}{2}\,\omega^{l\;m}{}_{;m}\,u_l\,u^a-\frac{1}{2}\,\omega^{l\;m}{}_{;l}\,u_m\,u^a+\frac{1}{2}\,\omega^{a\;m}{}_{;m}-\frac{1}{2}\,\omega^{l\;a}{}_{;l}-\frac{1}{2}\,\omega^{a\;m}\,u_k\,u^k{}_{;m}+\frac{1}{2}\,\omega^{l\;a}\,u_k\,u^k{}_{;l}=0$$

> *proof*[26 *e*] := *expand*(*TEDS*(u[-*l*]·omega[*l*, *a*] = 0, *proof*[26 *d*])) : *T*(%);

$$\frac{1}{2}\,\omega^{a\;m}{}_{;k}\,u_m\,u^k-\frac{1}{2}\,\omega^{l\;a}{}_{;k}\,u_l\,u^k+\frac{2}{3}\,P^a{}_b\,\theta^{;b}-\frac{1}{2}\,\omega^{l\;m}\,u_m\,u^a{}_{;l}+\frac{1}{2}\,\omega^{a\;m}\,u_m\,u^k{}_{;k}+\frac{1}{2}\,\omega^{l\;m}{}_{;m}\,u_l\,u^a-\frac{1}{2}\,\omega^{l\;m}{}_{;l}\,u_m\,u^a+\frac{1}{2}\,\omega^{a\;m}{}_{;m}-\frac{1}{2}\,\omega^{l\;a}{}_{;l}-\frac{1}{2}\,\omega^{a\;m}\,u_k\,u^k{}_{;m}+\frac{1}{2}\,\omega^{l\;a}\,u_k\,u^k{}_{;l}=0 \quad \textbf{(1.14)}$$

> *proof*[26 *f*] := *expand*(*TEDS*(u[-*m*]·omega[*l*, *m*] = 0, *proof*[26 *e*])) : *T*(%);

$$\frac{1}{2}\,\omega^{a\;m}{}_{;k}\,u_m\,u^k-\frac{1}{2}\,\omega^{l\;a}{}_{;k}\,u_l\,u^k+\frac{2}{3}\,P^a{}_b\,\theta^{;b}+\frac{1}{2}\,\omega^{a\;m}\,u_m\,u^k{}_{;k}+\frac{1}{2}\,\omega^{l\;m}{}_{;m}\,u_l\,u^a-\frac{1}{2}\,\omega^{l\;m}{}_{;l}\,u_m\,u^a+\frac{1}{2}\,\omega^{a\;m}{}_{;m}-\frac{1}{2}\,\omega^{l\;a}{}_{;l}-\frac{1}{2}\,\omega^{a\;m}\,u_k\,u^k{}_{;m}+\frac{1}{2}\,\omega^{l\;a}\,u_k\,u^k{}_{;l}=0 \quad \textbf{(1.15)}$$

> *proof*[26 *g*] := *expand*(*TEDS*(u[-*k*]·u[k, -L] = 0, *proof*[26 *f*])) : *T*(%);

$$\frac{1}{2}\,\omega^{a\;m}{}_{;k}\,u_m\,u^k-\frac{1}{2}\,\omega^{l\;a}{}_{;k}\,u_l\,u^k+\frac{2}{3}\,P^a{}_b\,\theta^{;b}+\frac{1}{2}\,\omega^{a\;m}\,u_m\,u^k{}_{;k}+\frac{1}{2}\,\omega^{l\;m}{}_{;m}\,u_l\,u^a-\frac{1}{2}\,\omega^{l\;m}{}_{;l}\,u_m\,u^a+\frac{1}{2}\,\omega^{a\;m}{}_{;m}-\frac{1}{2}\,\omega^{l\;a}{}_{;l}-\frac{1}{2}\,\omega^{a\;m}\,u_k\,u^k{}_{;m}=0 \quad \textbf{(1.16)}$$

> *proof*[26 *g2*] := *expand*(*TEDS*(u[-*k*]·u[k, -M] = 0, *proof*[26 *g*])) : *T*(%);

$$\frac{1}{2}\,\omega^{a\;m}{}_{;k}\,u_m\,u^k-\frac{1}{2}\,\omega^{l\;a}{}_{;k}\,u_l\,u^k+\frac{2}{3}\,P^a{}_b\,\theta^{;b}+\frac{1}{2}\,\omega^{a\;m}\,u_m\,u^k{}_{;k}+\frac{1}{2}\,\omega^{l\;m}{}_{;m}\,u_l\,u^a-\frac{1}{2}\,\omega^{l\;m}{}_{;l}\,u_m\,u^a+\frac{1}{2}\,\omega^{a\;m}{}_{;m}-\frac{1}{2}\,\omega^{l\;a}{}_{;l}=0 \quad \textbf{(1.17)}$$

> *proof*[26 *g3*] := *expand*(*TEDS*(u[-*m*]·omega[a, m] = 0, *proof*[26 *g2*])) : *T*(%);

$$\frac{1}{2}\,\omega^{a\;m}{}_{;k}\,u_m\,u^k-\frac{1}{2}\,\omega^{l\;a}{}_{;k}\,u_l\,u^k+\frac{2}{3}\,P^a{}_b\,\theta^{;b}+\frac{1}{2}\,\omega^{l\;m}{}_{;m}\,u_l\,u^a-\frac{1}{2}\,\omega^{l\;m}{}_{;l}\,u_m\,u^a+\frac{1}{2}\,\omega^{a\;m}{}_{;m}-\frac{1}{2}\,\omega^{l\;a}{}_{;l}=0 \quad \textbf{(1.18)}$$

> *proof*[26 *g4*] := *expand*(*TEDS*(u[-*m*]·omega[a, m, -K] =-u[-*l*]·omega[l, a, -K], *proof*[26 *g3*])) : *T*(%);

$$-\omega^{l\;a}{}_{;k}\,u_l\,u^k+\frac{2}{3}\,P^a{}_b\,\theta^{;b}+\frac{1}{2}\,\omega^{l\;m}{}_{;m}\,u_l\,u^a-\frac{1}{2}\,\omega^{l\;m}{}_{;l}\,u_m\,u^a+\frac{1}{2}\,\omega^{a\;m}{}_{;m} \quad \textbf{(1.19)}$$

$$-\frac{1}{2}\,\omega^{l\ a}{}_{;l}=0$$

```
> proof[26 g5] := expand(TEDS(u[-m]·omega[l, m, -L] =-u[-l]·omega[l, m, -M],
    proof[26 g4])) : T(%);
```

$$-\omega^{l\ a}{}_{;k}u_l u^k+\frac{2}{3}P^a{}_b\theta^{;b}+\omega^{l\ m}{}_{;m}u_l u^a+\frac{1}{2}\omega^{a\ m}{}_{;m}-\frac{1}{2}\omega^{l\ a}{}_{;l}=0 \quad (1.20)$$

```
> proof[26 g6] := expand(TEDS(omega[l, a, -L] =-omega[a, m, -M], proof[26 g5])) : T(%);
```

$$-\omega^{l\ a}{}_{;k}u_l u^k+\frac{2}{3}P^a{}_b\theta^{;b}+\omega^{l\ m}{}_{;m}u_l u^a+\omega^{a\ m}{}_{;m}=0 \quad (1.21)$$

from vid30:

```
> proof[26 g7] := expand(TEDS(u[-1]·u[k]·omega[l, a, -K] = 0, proof[26 g6])) : T(%);
```

$$\frac{2}{3}P^a{}_b\theta^{;b}+\omega^{l\ m}{}_{;m}u_l u^a+\omega^{a\ m}{}_{;m}=0 \quad (1.22)$$

```
> proof[26 g8] := 3· subs(l = b, m = d, M = D, proof[26 g7]) : T(%);
```

$$3\,\omega^{b\ d}{}_{;d}u_b u^a+2P^a{}_b\theta^{;b}+3\,\omega^{a\ d}{}_{;d}=0 \quad (1.23)$$

```
>
>
>
>
```

Now we compare to the full version of eq26 with acceleration=0:

```
> eq[26] := 2·P[a, b]·theta[-B] + 3·P[a, -b]·omega[b, d, -D] : T(%);
```

$$2P^{a\ b}\theta_{;b}+3P^a{}_b\omega^{b\ d}{}_{;d} \quad (1.24)$$

```
> temp := subs(b =-b, B =-B, eq[26]);
```

$$temp := 3P_{a,b}\omega_{-b,d,-D}+2P_{a,-b}\theta_B \quad (1.25)$$

```
> proof[26 h] := expand(TEDS(P[a, b] = g[a, b] + u[a]·u[b], temp)) : T(%);
```

$$3\,\omega_b{}^d{}_{;d}u^a u^b+2P^a{}_b\theta^{;b}+3g^{a\ b}\omega_b{}^d{}_{;d} \quad (1.26)$$

```
> Absorbg(proof[26 h]) : T(%);
```

$$3\,\omega_b{}^d{}_{;d}u^a u^b+2P^a{}_b\theta^{;b}+3\,\omega^{a\ d}{}_{;d} \quad (1.27)$$

```
> proof[26 g8] : T(%);
```

$$3\,\omega^{b\ d}{}_{;d}u_b u^a+2P^a{}_b\theta^{;b}+3\,\omega^{a\ d}{}_{;d}=0 \quad (1.28)$$

which is identical to *proof*[26 *g8*] above

proven

****
****

****

Equation 27 (from literature)

****

```
> eq[27] := omega[a,-A]=2·du[a]·omega[-a] : T(%);
```

$$\omega^{a}{}_{;a} = 2\, du^{a}\, \omega_{a} \tag{1.29}$$

the following 2 statements are equivalent for dust:

```
> eq[27 a] := TEDS(du[a]=0, eq[27]) : T(%);
```

$$\omega^{a}{}_{;a} = 0 \tag{1.30}$$

```
> eq[27 b] := eta[a, b, c, d]·u[-a]·omega[-c,-d,-B]=0 : T(%);
```

$$\eta^{a\,b\,c\,d}\, u_{a}\, \omega_{c\,d\,;b} = 0 \tag{1.31}$$

****

Equation 28 - identity of the magnetic part of the Weyl Tensor (in literature)

****

```
> eq[28] := H[-a,-b]=symm(P[-a, c]·P[-b, d]·omega[d, C],-a,-b) : T(%);
```

$$H_{a\,b} = \frac{1}{2}\, P_{a}{}^{c}\, P_{b}{}^{d}\, \omega^{d\;;c} + \frac{1}{2}\, P_{b}{}^{c}\, P_{a}{}^{d}\, \omega^{d\;;c} \tag{1.32}$$

****
****

```
> save eq, "Seneqs2d" :
```

go to page 3

```
> read "Seneqs2d" :
> PrintSubArray(eq, 1, 28, y);
```

$$1,\; T_{a\,b} = \rho\, u_{a}\, u_{b}$$

$$2,\; P_{a\,b} = u\, u_{b\,a} + g_{a\,b}$$

$$3,\; P^{a}{}_{b}\, u^{b} = 0$$

$$4,\; dX^{a} = u^{b}\, X^{a}{}_{;b}$$

$$5,\; du^{a} = u^{b}\, u^{a}{}_{;b}$$

$$6,\; u_{a\,;b} = \frac{1}{3}\,\theta\, P_{a\,b} + \sigma_{a\,b} + \omega_{a\,b} - du_{a}\, u_{b}$$

$$7,\; \theta = u^{a}{}_{;a}$$

$$8,\; \sigma_{a\,b} = \frac{1}{2}\, P_{a}{}^{c}\, P_{b}{}^{d}\, u_{c\,;d} + \frac{1}{2}\, P_{b}{}^{c}\, P_{a}{}^{d}\, u_{c\,;d} - \frac{1}{3}\,\theta\, P_{a\,b}$$

$$9,\; \omega_{a\,b} = \frac{1}{2}\, P_{a}{}^{c}\, P_{b}{}^{d}\, u_{c\,;d} - \frac{1}{2}\, P_{b}{}^{c}\, P_{a}{}^{d}\, u_{c\,;d}$$

$$10,\; \omega^{a} = \frac{1}{2}\,\eta^{a\,b\,c\,d}\, u_{b}\, \omega_{c\,d}$$

$$11,\; \omega_{a\,b} = \eta_{a\,b\,e\,f}\, \omega^{e}\, u^{f}$$

$$12, \omega^2 = \frac{1}{2}\,\omega^{a\,b}\,\omega_{a\,b}$$

13, "iff(iff(omega[-a,-b] = 0,omega[-a]),omega = 0)"

$$14, \omega_a{}^c\,\omega_c{}^b = -\omega^2 P_a{}^b + \omega^b\,\omega_a$$

$$15, \frac{1}{2}\,u_{b\,;a} - \frac{1}{2}\,u_{a\,;b} = \frac{1}{2}\,du_a\,u_b - \frac{1}{2}\,du_b\,u_a + \omega^{a\,b}$$

$$16, -\frac{1}{6}\,u_c\,u_{a\,;b} + \frac{1}{6}\,u_c\,u_{b\,;a} + \frac{1}{6}\,u_b\,u_{a\,;c} - \frac{1}{6}\,u_b\,u_{c\,;a} - \frac{1}{6}\,u_a\,u_{b\,;c} + \frac{1}{6}\,u_a\,u_{c\,;b} = 0$$

$$17, \sigma_{a\,b} = 0$$

$$18, u_{b\,;a} = -u_a\,u_{b\,;c}\,u^c + \frac{1}{3}\,\theta\,h_{a\,b} + \omega_{a\,b}$$

$$19, u^a{}_{;c\,;d} - u^a{}_{;d\,;c} = R^a{}_{b\,c\,d}\,u^b$$

$$20, dottheta + \frac{1}{3}\,\theta^2 - 2\,\omega^2 + \frac{1}{2}\,\mu = 0$$

$$21, P_a{}^c\,P_b{}^d\,\omega_{c\,d\,;f}\,u^f + \frac{2}{3}\,\theta\,\omega_{a\,b} = 0$$

$$22, \omega_a\,\omega_b - \frac{1}{3}\,P_{a\,b}\,\omega^2 + E_{a\,b} = 0$$

$$23, E_{a\,b} = C_{a\,b\,c\,d}\,u^c\,u^d$$

$$24, H_{a\,b} = \frac{1}{2}\,\eta_{a\,e}{}^{c\,d}\,C_{c\,d\,b\,f}\,u^e\,u^f$$

$$25, P^a{}_b\,\omega^b{}_{;f}\,u^f + \frac{2}{3}\,\theta\,\omega^a = 0$$

$$26, 2\,P^{a\,b}\,\theta_{;b} + 3\,P^a{}_b\,\omega^{b\,d}{}_{;d}$$

$$27, \omega^a{}_{;a} = 2\,du^a\,\omega_a$$

$$28, H_{a\,b} = \frac{1}{2}\,P_a{}^c\,P_b{}^d\,\omega^{d\;;c} + \frac{1}{2}\,P_b{}^c\,P_a{}^d\,\omega^{d\;;c} \qquad \textbf{(1.33)}$$

>

>