```
> restart;with(Riemann):with(Canon):with(TensorPack): CDF(0); CDS(index):
> read "EFE": read "SFE":read "fids":read "seneqs80":
```

Chapter XX
Using Ricci Identities
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Riemann tensor (file 3): contraction with u[d]
SSSeq81- time differentation of SSSeq79

In this file we aim to obtain an expression for the time differentation of SSSeq79.

```
> eq[79] := (mu + p)·ω^2 + W[a, b]·W[-a,-b] = 0 : T(%);
```

$$(\mu + p)\,\omega^2 + W^{a\ b}\,W_{a\ b} = 0 \tag{1.1}$$

where

```
> temp1 := W[-a,-b] = symm(Q[c,-a]·Q[-b, d]·omega[-d,-C],-a,-b) - 1/2·Q[-a,-b]
      ·Q[c, d]·omega[-d,-C] : T(%);
```

$$W_{a\ b} = \frac{1}{2}\,Q^{c}{}_{a}\,Q_{b}{}^{d}\,\omega_{d\ ;c} + \frac{1}{2}\,Q^{c}{}_{b}\,Q_{a}{}^{d}\,\omega_{d\ ;c} - \frac{1}{2}\,Q_{a\ b}\,Q^{c\ d}\,\omega_{d\ ;c} \tag{1.2}$$

and

```
> temp2a := Q[-a,-b] = omega[-a, c]·omega[-b,-c] / ω^2 : T(%);
```

$$Q_{a\ b} = \frac{\omega_{a}{}^{c}\,\omega_{b\ c}}{\omega^2} \tag{1.3}$$

Note that Q is the projector orthogonal to both u and omega
We use the identity eq14:

```
> temp2b := subs(b = -b, -TEDS(omega[-c, b] = -omega[b,-c], eq[14])) : T(%);
```

$$\omega_{a}{}^{c}\,\omega_{b\ c} = P\,\omega^2{}_{a\ b} - \omega\,\omega_{a\ b} \tag{1.4}$$

```
> temp2 := expand(TEDS(temp2b, temp2a)) : T(%);
```

$$Q_{a\ b} = P_{a\ b} - \frac{\omega_{a}\,\omega_{b}}{\omega^2} \tag{1.5}$$

```
>
```

and furthermore

```
>
> eq[80] := dotQ[-a,-b] = 0 : T(%);
```

$$dotQ_{a\ b} = 0 \tag{1.6}$$

Firstly

```
> temp3 := dotT(eq[79]) : T(%);
```

(1.7)

$$(dotmu + dotp)\,\omega^2 + 2\,(\mu + p)\,\omega\,dotomega + W^{a\ b}\,dotW_{a\,b} + dotW^{a\ b}\,W_{a\,b} = 0 \qquad (1.7)$$

so we need an expression for dotW[a,b]

```
> temp4 := TEDS(dotQ[c, d] = 0, TEDS(dotQ[-a, -b] = 0, TEDS(dotQ[-a, d] = 0,
      TEDS(dotQ[c, -b] = 0, TEDS(dotQ[-b, d] = 0, TEDS(dotQ[c, -a] = 0,
      dotT(temp1)))))));
```

$$dotW_{a\,b} = \frac{1}{2}\,Q^{c}{}_{a}\,Q_{b}{}^{d}\,dotomega_{d\,;c} + \frac{1}{2}\,Q^{c}{}_{b}\,Q_{a}{}^{d}\,dotomega_{d\,;c} \qquad (1.8)$$

$$-\frac{1}{2}\,Q_{a\,b}\,Q^{c\ d}\,dotomega_{d\,;c}$$

```
>
```

from kinematic quantities:

```
> temp5 := dotomega[-a, -C] = 1/9 * theta^2 * omega[-c] * u[-a] + 1/3 * theta * du[-f]
      * omega[f] * u[-a] * u[-c] - du[-c] * du[-f] * omega[f] * u[-a] + 1/3 * P[-c, -a]
      * omega[f] * theta[-F] - 1/3 * theta[-A] * omega[-c] - du[-c, -F] * omega[f] * u[-a]
      + omega[f] * omega[-c, -a, -F] - omega[f] * omega[-c, -f, -A] + u[d] * omega[-a, -D,
      -C] : T(%);
```

$$dotomega_{a\,;c} = \frac{1}{9}\,\theta^2\,\omega_c\,u_a + \frac{1}{3}\,\theta\,du_f\,\omega^f\,u_a\,u_c - du_c\,du_f\,\omega^f\,u_a + \frac{1}{3}\,P_{c\,a}\,\omega^f\,\theta_{;f} \qquad (1.9)$$

$$-\frac{1}{3}\,\theta_{;a}\,\omega_c - du_{c\,;f}\,\omega^f\,u_a + \omega^f\,\omega_{c\,a\,;f} - \omega^f\,\omega_{c\,f\,;a} + u^d\,\omega_{a\,;d\,;c}$$

```
> temp6 := expand(TEDS(subs(b = -d, temp2), TEDS(subs(a = -c, temp2), TEDS(subs(b = -d,
      a = b, temp2), TEDS(subs(a = -c, b = a, temp2), TEDS(subs(a = -c, b = -d, temp2),
      expand(TELS(temp2, temp4)))))))) : T(%);
```

$$dotW_{a\,b} = -\frac{1}{2}\,dotomega_{d\,;c}\,P^{c\ d}\,P_{a\,b} + \frac{1}{2}\,dotomega_{d\,;c}\,P^{c}{}_{a}\,P_{b}{}^{d} \qquad (1.10)$$

$$+\frac{1}{2}\,dotomega_{d\,;c}\,P^{c}{}_{b}\,P_{a}{}^{d} + \frac{1}{2}\,\frac{dotomega_{d\,;c}\,P^{c\ d}\,\omega_a\,\omega_b}{\omega^2}$$

$$-\frac{1}{2}\,\frac{dotomega_{d\,;c}\,P^{c}{}_{a}\,\omega^{d}\,\omega_b}{\omega^2} - \frac{1}{2}\,\frac{dotomega_{d\,;c}\,P^{c}{}_{b}\,\omega^{d}\,\omega_a}{\omega^2}$$

$$-\frac{1}{2}\,\frac{dotomega_{d\,;c}\,P_{a}{}^{d}\,\omega^{c}\,\omega_b}{\omega^2} + \frac{1}{2}\,\frac{dotomega_{d\,;c}\,P_{a\,b}\,\omega^{c}\,\omega^{d}}{\omega^2}$$

$$-\frac{1}{2}\,\frac{dotomega_{d\,;c}\,P_{b}{}^{d}\,\omega^{c}\,\omega_a}{\omega^2} + \frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega^{c}\,\omega^{d}\,\omega_a\,\omega_b}{\omega^4}$$

```
>
```

```
> temp7 := Absorbg(TELS(P[-c, -d] = g[-c, -d] + u[-c]·u[-d], expand(TELS(P[c, d]
      = g[c, d] + u[c]·u[d], expand(temp6))))) : T(%);
```

$$dotW_{a\,b} = \frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega^{c}\,\omega^{d}\,\omega_a\,\omega_b}{\omega^4} + \frac{1}{2}\,u^{c}\,u^{d}\,u_a\,u_b\,dotomega_{d\,;c} \qquad (1.11)$$

$$+\frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega^{c}\,\omega^{d}\,g_{a\,b}}{\omega^{2}}-\frac{1}{2}\,\frac{dotomega_{b\,;c}\,\omega^{c}\,\omega_{a}}{\omega^{2}}$$

$$+\frac{1}{2}\,\frac{dotomega_{d}^{\;;d}\,\omega_{a}\,\omega_{b}}{\omega^{2}}-\frac{1}{2}\,\frac{dotomega_{d\,;a}\,\omega^{d}\,\omega_{b}}{\omega^{2}}-\frac{1}{2}\,\frac{dotomega_{d\,;b}\,\omega^{d}\,\omega_{a}}{\omega^{2}}$$

$$-\frac{1}{2}\,\frac{dotomega_{a\,;c}\,\omega^{c}\,\omega_{b}}{\omega^{2}}+\frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega_{a}\,\omega_{b}\,u^{c}\,u^{d}}{\omega^{2}}$$

$$-\frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega^{d}\,\omega_{b}\,u^{c}\,u_{a}}{\omega^{2}}-\frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega^{d}\,\omega_{a}\,u^{c}\,u_{b}}{\omega^{2}}$$

$$-\frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega^{c}\,\omega_{b}\,u^{d}\,u_{a}}{\omega^{2}}+\frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega^{c}\,\omega^{d}\,u_{a}\,u_{b}}{\omega^{2}}$$

$$-\frac{1}{2}\,\frac{dotomega_{d\,;c}\,\omega^{c}\,\omega_{a}\,u^{d}\,u_{b}}{\omega^{2}}+\frac{1}{2}\,u^{c}\,u_{b}\,dotomega_{a\,;c}$$

$$+\frac{1}{2}\,u^{d}\,u_{a}\,dotomega_{d\,;b}-\frac{1}{2}\,g_{a\,b}\,u^{c}\,u^{d}\,dotomega_{d\,;c}-\frac{1}{2}\,u_{a}\,u_{b}\,dotomega_{d}^{\;;d}$$

$$+\frac{1}{2}\,u^{c}\,u_{a}\,dotomega_{b\,;c}+\frac{1}{2}\,u^{d}\,u_{b}\,dotomega_{d\,;a}-\frac{1}{2}\,g_{a\,b}\,dotomega_{d}^{\;;d}$$

$$+\frac{1}{2}\,g_{b}^{\;d}\,dotomega_{d\,;a}+\frac{1}{2}\,g_{a}^{\;d}\,dotomega_{d\,;b}$$

>

incomplete

>

>