```
> restart;with(Riemann):with(Canon):with(TensorPack): CDF(0); CDS(index):
> read "EFE": read "SFE":read "fids":read "seneqs80":
```

Chapter XX
Using Ricci Identities
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Riemann tensor (file 3): contraction with u[d]
SSSeq80 - completed

> $eq[80] := dotQ[-a,-b] = 0 : T(\%);$

$$dotQ_{a\,b} = 0 \quad \textbf{(1.1)}$$

where

> $defn := Q[-a,-b] = \frac{\text{omega}[-a,c]\cdot\text{omega}[-b,-c]}{\omega^2} : T(\%);$

$$Q_{a\,b} = \frac{\omega_a{}^c\,\omega_{b\,c}}{\omega^2} \quad \textbf{(1.2)}$$

Note that Q is easily seen as the the projector orthogonal to both u and omega

>

> $temp2 := dotT(defn) : T(\%);$

$$dotQ_{a\,b} = \frac{dotomega_a{}^c\,\omega_{b\,c}}{\omega^2} + \frac{\omega_a{}^c\,dotomega_{b\,c}}{\omega^2} - \frac{2\,\omega_a{}^c\,\omega_{b\,c}\,dotomega}{\omega^3} \quad \textbf{(1.3)}$$

>

$dotomega_{a\,b} = \theta\,\omega_{a\,b}\,p' - \frac{2}{3}\,\theta\,\omega_{a\,b} + u_a\,du^c\,\omega_{c\,b} + u_b\,du^d\,\omega_{a\,d}$

>

> $temp3 := dotomega[-a,-b] = \text{theta}\cdot\omega[-a,-b]\cdot`p'` - \frac{2}{3}\cdot\theta\cdot\omega[-a,-b] + u[-a]\cdot du[c]\cdot\omega[-c,-b] + u[-b]\cdot du[d]\cdot\omega[-a,-d] : T(\%);$

$$dotomega_{a\,b} = p'\,\theta\,\omega_{a\,b} - \frac{2}{3}\,\theta\,\omega_{a\,b} + u_a\,du^c\,\omega_{c\,b} + u_b\,du^d\,\omega_{a\,d} \quad \textbf{(1.4)}$$

with the assumptions:

> $temp3a := TEDS(du[d]\cdot\text{omega}[-a,-d] = 0, TEDS(du[c]\cdot\text{omega}[-c,-b] = 0, temp3)) : T(\%);$

$$dotomega_{a\,b} = p'\,\theta\,\omega_{a\,b} - \frac{2}{3}\,\theta\,\omega_{a\,b} \quad \textbf{(1.5)}$$

>

> $temp4 := expand(TEDS(subs(b=-c, temp3a), temp2)) : T(\%);$

(1.6)

$$dotQ_{a\,b} = \frac{\omega_a{}^c\,\omega_{b\,c}\,\theta\,p'}{\omega^2} - \frac{2}{3}\,\frac{\omega_a{}^c\,\omega_{b\,c}\,\theta}{\omega^2} + \frac{\omega_a{}^c\,dotomega_{b\,c}}{\omega^2} - \frac{2\,\omega_a{}^c\,\omega_{b\,c}\,dotomega}{\omega^3} \tag{1.6}$$

> $temp5 := expand(TEDS(subs(b = c, a = b, temp3a), temp4)) : T(\%);$

$$dotQ_{a\,b} = \frac{2\,\omega_a{}^c\,\omega_{b\,c}\,\theta\,p'}{\omega^2} - \frac{4}{3}\,\frac{\omega_a{}^c\,\omega_{b\,c}\,\theta}{\omega^2} - \frac{2\,\omega_a{}^c\,\omega_{b\,c}\,dotomega}{\omega^3} \tag{1.7}$$

> $temp6 := dotomega = \theta\cdot\omega\cdot `p'` - \frac{2}{3}\cdot\theta\cdot\omega : T(\%);$

$$dotomega = \theta\,p'\,\omega - \frac{2}{3}\,\theta\,\omega \tag{1.8}$$

> $temp7 := expand(TEDS(temp6, temp5)) : T(\%);$

$$dotQ_{a\,b} = 0 \tag{1.9}$$

proof completed