```
> restart;with(Riemann):with(TensorPack): with(Canon):CDF(0): CDS(index):
```

Chapter XX Tensor analysis using indices - Senovilla et al. - Shearfree for acceleration parallel to vorticity if $\sigma_{a\,b}$=0 =>ωΘ=0

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eq78 - attempt intially to prove SSSeq74a directly to SSSeq78 - almost

```
> read "EFE" : read "SFE" :read "fids" :read "Seneqs80" :
> eq[74 b] := omega[a, b]·omega[c]·omega[-c,-B] = 0 : T(%);
```

$$\omega^{a\,b}\,\omega^{c}\,\omega_{c\,;b} = 0 \qquad (1.1)$$

```
> convert(eq[68], string);
```

"1/2*P[-a,-b]*omega[c]*omega[-b,-C]-1/2*P[-a,-b]*omega[c]*omega[-c,-B] = 0" (1.2)

```
> eq[68] := 1/2*P[a, b]*omega[c]*omega[-b,-C]-1/2*P[a, b]*omega[c]*omega[-c,
  -B] = 0 : T(%);
```

$$\frac{1}{2}\,P^{a\,b}\,\omega^{c}\,\omega_{b\,;c} - \frac{1}{2}\,P^{a\,b}\,\omega^{c}\,\omega_{c\,;b} = 0 \qquad (1.3)$$

```
>
```

Now combining SSSeqs68 and 74b

```
> temp1 := subs(d = a, TEDS(omega[b, d] =-omega[d, b], subs(d =-d, TEDS(omega[-a,-d]
  ·P[a, b] = omega[b,-d], expand(omega[-a,-d]·eq[68]))))) : T(%);
```

$$-\frac{1}{2}\,\omega^{c}\,\omega_{b\,;c}\,\omega^{a\,b} + \frac{1}{2}\,\omega^{a\,b}\,\omega^{c}\,\omega_{c\,;b} = 0 \qquad (1.4)$$

```
> temp2 := -2 TEDS(eq[74 b], temp1) : T(%);
```

$$\omega^{c}\,\omega_{b\,;c}\,\omega^{a\,b} = 0 \qquad (1.5)$$

```
>
```

Now we contract with omega[-a,d]:

```
> temp3 := omega[-a, d]·temp2 : T(%);
```

$$\omega_{a}^{\;\;d}\,\omega^{c}\,\omega_{b\,;c}\,\omega^{a\,b} = 0 \qquad (1.6)$$

Now using SSSeq14:

```
> temp4 := subs(a =-d, c =-a, TEDS(omega[-a, c] =-omega[c,-a], eq[14])) : T(%);
```

$$-\omega^{a\,b}\,\omega_{a}^{\;\;d} = -\omega^{2}\,P^{d\,b} + \omega^{b}\,\omega^{d} \qquad (1.7)$$

```
> temp5 := expand(TEDS(temp4, temp3)) : T(%);
```

$$\omega^{2}\,P^{d\,b}\,\omega^{c}\,\omega_{b\,;c} - \omega^{b}\,\omega^{c}\,\omega^{d}\,\omega_{b\,;c} = 0 \qquad (1.8)$$

and also SSSeqs68 again:

```
> temp6 := TEDS(P[a, b]·omega[-a] = omega[b], 2· expand(omega[-a]·eq[68])) : T(%);
```

$$\omega^{c}\,\omega_{b\,;c}\,\omega^{b} - \omega^{c}\,\omega_{c\,;b}\,\omega^{b} = 0 \qquad (1.9)$$

> temp7 := Absorbg(TEDS(P[d, b] = g[d, b] + u[d]·u[b], temp5)) : T(%);

0, "not a tensor"

$$\omega^2\,\omega^{c}\,\omega_{b\,;c}\,u^{b}\,u^{d} + \omega^2\,\omega^{c}\,\omega^{d}{}_{;c} - \omega^{b}\,\omega^{c}\,\omega^{d}\,\omega_{b\,;c} = 0 \quad \textbf{(1.10)}$$

We look at the term $\omega_{b\,;c}\,u^{b}$

> temp8 := TEDS(du[b]·omega[-b] = Psi·omega·omega, expand(TEDS(omega[-b]·omega[b, -c] = 0, expand(TEDS(subs(sigma = 0, b = c, B = C, a = -b, eq[6]), omega[-b, -C]·u[b] = omega[-b]·u[b, -C]))))) : T(%);

$$\omega_{b\,;c}\,u^{b} = -u_{c}\,\Psi\,\omega^2 + \frac{1}{3}\,\theta\,P^{b}{}_{c}\,\omega_{b} \quad \textbf{(1.11)}$$

which is substituted back into temp7:

> temp9 := expand(TEDS(omega[c]·omega[-c] = ω^2, expand(TEDS(omega[c]·u[-c] = 0, TEDS(P[b, -c]·omega[-b] = omega[-c], TEDS(temp8, temp7)))))) : T(%);

$$\omega^2\,\omega^{c}\,\omega^{d}{}_{;c} - \omega^{b}\,\omega^{c}\,\omega^{d}\,\omega_{b\,;c} + \frac{1}{3}\,\omega^4\,u^{d}\,\theta = 0 \quad \textbf{(1.12)}$$

<u>we are almost at SSSeq78 with this expression (temp9).</u>

> eq[74] := `p'`/Psi ·omega[a]·omega[-A] = (`p'` − 2/3)·ω^4 : T(%);

$$\frac{p'\,\omega^{a}\,\omega_{;a}}{\Psi} = \left(p' - \frac{2}{3}\right)\omega^4 \quad \textbf{(1.13)}$$

Now

> temp10 := omega[b]·omega[-b, -C] = 1/2 ·omega[-C] : T(%)

$$\omega^{b}\,\omega_{b\,;c} = \frac{1}{2}\,\omega_{;c} \quad \textbf{(1.14)}$$

> temp11 := TEDS(temp10, temp9) : T(%);

$$\omega^2\,\omega^{c}\,\omega^{d}{}_{;c} - \frac{1}{2}\,\omega^{c}\,\omega^{d}\,\omega_{;c} + \frac{1}{3}\,\omega^4\,u^{d}\,\theta = 0 \quad \textbf{(1.15)}$$

> temp12 := subs(a = c, A = C, eq[74]·Psi / `p'`) : T(%);

$$\omega^{c}\,\omega_{;c} = \frac{\Psi\left(p' - \frac{2}{3}\right)\omega^4}{p'} \quad \textbf{(1.16)}$$

> temp13 := collect(isolate(expand(TEDS(temp12, temp11) / ω^2), omega[c]·omega[d, -C]), [ω^2, ω[d]]) : T(%);

$$\omega^{c}\,\omega^{d}{}_{;c} = \left(\left(\frac{1}{2}\,\Psi - \frac{1}{3}\,\frac{\Psi}{p'}\right)\omega^{d} - \frac{1}{3}\,u^{d}\,\theta\right)\omega^2 \quad \textbf{(1.17)}$$

>

proof completed