```
> restart;with(Riemann):with(Canon):with(TensorPack): CDF(0); CDS(index):
> read "EFE": read "SFE":read "fids":read "seneqs80":
```

**Chapter XX**
**Using Ricci Identities**
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**Riemann tensor (file 3): contraction with u[d]**
**SSSeq77**

```
> eq[77] := R[-b,-d]·omega[b]·omega[d] = `cod(omega[b]*omega[c,-B],-c)` - omega[b, C]
      ·omega[-c,-B] - omega[b]·omega[c,-C,-B] : T(%);
```

$$R_{bd}\,\omega^{b}\,\omega^{d} = -\omega^{b}\,\omega^{c}{}_{;c\,;b} - \omega^{b\,;c}\,\omega_{c\,;b} + cod(omega[b]*omega[c,-B],-c) \quad (1.1)$$

Now a definition of the Riemann tensor (Ellis 1970) OR (Kay 1988) is:

```
> temp := R[-a,-b,-c,-d]·u[a] = u[-b,-C,-D] - u[-b,-D,-C] : T(%);
```

$$R_{abcd}\,u^{a} = u_{b\,;c\,;d} - u_{b\,;d\,;c} \quad (1.2)$$

For the omega vector

```
> temp1 := R[-a,-b,-c,-d]·omega[a] = omega[-b,-C,-D] - omega[-b,-D,-C] : T(%);
```

$$R_{abcd}\,\omega^{a} = \omega_{b\,;c\,;d} - \omega_{b\,;d\,;c} \quad (1.3)$$

rearranging:

```
> temp2 := subs(d=f, D=F, c=g, C=G, a=d, b=c, g=a, f=b, G=A, F=B, temp1) : T(%);
```

$$R_{dcab}\,\omega^{d} = \omega_{c\,;a\,;b} - \omega_{c\,;b\,;a} \quad (1.4)$$

which allowing for the antisymmetry of the first 2 indices of the Riemann tensor, is the first part of SSSeq77

```
>
```

Now looking at the LHS of the second part of SSSeq77, we have

```
> temp3 := cod(omega[b]·omega[c,-B],-c) - omega[b, C]·omega[-c,-B] - omega[b]
      ·omega[c,-C,-B] : T(%);
```

$$\omega^{b}\,\omega^{c}{}_{;b\,;c} - \omega^{b}\,\omega^{c}{}_{;c\,;b} - \omega^{b\,;c}\,\omega_{c\,;b} + \omega^{b}{}_{;c}\,\omega^{c}{}_{;b} \quad (1.5)$$

We can see that that the last 2 terms cancel i.e.:

```
> temp4 := TEDS(omega[b, C]·omega[-c,-B] = omega[b,-C]·omega[c,-B], temp3) : T(%);
```

$$\omega^{b}\,\omega^{c}{}_{;b\,;c} - \omega^{b}\,\omega^{c}{}_{;c\,;b} \quad (1.6)$$

remembering that this is the LHS of SSSeq77b

```
>
>
```

Now contracting temp2 wrt a and c, tp form the Ricci tensor with symmetry change::

```
> temp5 := -TEDS(R[-d, a,-a,-b] =-R[-d,-b], subs(c=-a, temp2)) : T(%);
```

**(1.7)**

$$\omega^{d} R_{d\,b} = -\omega^{a}{}_{;a\,;b} + \omega^{a}{}_{;b\,;a} \tag{1.7}$$

rearranging and contracting with omega[b]:

```
> temp6 := expand(omega[b]·subs(a = c, A = C, temp5)) : T(%);
```

$$\omega^{b}\,\omega^{d} R_{d\,b} = \omega^{b}\,\omega^{c}{}_{;b\,;c} - \omega^{b}\,\omega^{c}{}_{;c\,;b} \tag{1.8}$$

and we see that the RHS of temp6 equals LHS of SSSeq77b, and since the Ricci tensor is symmetric, we have proven SSSeq77b

end of proof

```
>
```

```
>
```