```
> restart;with(Riemann):with(TensorPack): with(Canon):CDF(0): CDS(index):
```

Chapter XX Tensor analysis using indices - Senovilla et al. - Shearfree for acceleration parallel to vorticity if $\sigma_{a\,b}$=0 =>ωΘ=0

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eq72c - omega[a,b]*cod(-a) contraction of SSSeq72

```
> read "EFE" : read "SFE" :read "fids" :read "Seneqs80" :
> SSSeq72 := ((3*`p'`^2/Psi^2 + 1/3)*theta^2-2*(Psi^2 + 1)*omega^2 + 1/2*mu + 3/2*p)*`p'`/Psi^2 = (3·(`p'`/Psi)^2 + 1/3 - PU·`p''`/`p'`)*omega^2 : T(%);
```

$$\frac{\left(\left(\frac{3\,p^2}{\Psi^2}+\frac{1}{3}\right)\theta^2-2\left(\Psi^2+1\right)\omega^2+\frac{1}{2}\mu+\frac{3}{2}p\right)p'}{\Psi^2}=\left(\frac{3\,p^2}{\Psi^2}+\frac{1}{3}-\frac{PU\,p''}{p'}\right)\omega^2 \qquad \textbf{(1.1)}$$

proof of eq72c: We commence with SSSeq72:

```
> temp := eq[72] : T(%);
```

$$\frac{\left(\left(\frac{3\,p^2}{\Psi^2}+\frac{1}{3}\right)\theta^2-2\left(\Psi^2+1\right)\omega^2+\frac{1}{2}\mu+\frac{3}{2}p\right)p'}{\Psi^2}=\left(\frac{3\,p^2}{\Psi^2}+\frac{1}{3}-\frac{PU\,p''}{p'}\right)\omega^2 \qquad \textbf{(1.2)}$$

```
> temp1 := expand(6·Ψ^4·`p'`·(expand(rhs(temp) - lhs(temp) = 0))) : T(%);
```

$$-6\,PU\,\Psi^4\omega^2p''+12\,\Psi^4\omega^2p^2+2\,\Psi^4\omega^2p'+18\,\Psi^2\omega^2p^3+12\,\Psi^2\omega^2p^2-2\,\Psi^2p^2\theta^2-18\,p^4\theta^2-3\,\Psi^2\mu\,p^2-9\,\Psi^2p\,p^2=0 \qquad \textbf{(1.3)}$$

taking the covariant derivative:

```
> temp2 := cod(temp1,-a) : T(%);
```

$$-36\,p^4\theta\,\theta_{;a}-3\,\Psi^2p^2\mu_{;a}-72\,p^3p'_{;a}\theta^2-9\,\Psi^2p^2p_{;a}+2\,\Psi^4p'_{;a}\omega^2-12\,\Psi^4\omega\,\omega_{;a}PU\,p''-24\,\Psi^3\Psi_{;a}\omega^2PU\,p''-6\,\Psi^4\omega^2PU\,p''_{;a}+24\,\Psi^4p'p'_{;a}\omega^2-6\,\Psi^4\omega^2PU_{;a}p''+24\,\Psi^4p^2\omega\,\omega_{;a}+48\,\Psi^3\Psi_{;a}p^2\omega^2+4\,\Psi^4p'\omega\,\omega_{;a}+8\,\Psi^3\Psi_{;a}p'\omega^2+54\,\Psi^2p^2p'_{;a}\omega^2+36\,\Psi^2p^3\omega\,\omega_{;a}+36\,\Psi\,\Psi_{;a}p^3\omega^2+24\,\Psi^2p'p'_{;a}\omega^2+24\,\Psi^2p^2\omega\,\omega_{;a}-4\,\Psi^2p^2\theta\,\theta_{;a}-4\,\Psi^2p'p'_{;a}\theta^2+24\,\Psi\,\Psi_{;a}p^2\omega^2-4\,\Psi\,\Psi_{;a}p^2\theta^2-6\,\Psi^2p'p'_{;a}\mu-18\,\Psi^2p'p'_{;a}p-6\,\Psi\,\Psi_{;a}p^2\mu-18\,\Psi\,\Psi_{;a}p^2p=0 \qquad \textbf{(1.4)}$$

and contract by omega[a,b]

```
> temp3 := expand(omega[a, b]·temp2) : T(%);
```

$$-4\,\Psi\,p^2\,\theta^2\,\Psi_{;a}\,\omega^{a\,b}+24\,\Psi^2\,\omega^2\,p'\,\omega^{a\,b}\,p'_{;a}+24\,\Psi^4\,\omega\,p^2\,\omega_{;a}\,\omega^{a\,b}$$
$$-18\,\Psi\,p\,p^2\,\Psi_{;a}\,\omega^{a\,b}-6\,\Psi^2\,\mu\,p'\,\omega^{a\,b}\,p'_{;a}-4\,\Psi^2\,p^2\,\theta\,\omega^{a\,b}\,\theta_{;a}$$
$$+36\,\Psi^2\,\omega\,p^3\,\omega_{;a}\,\omega^{a\,b}+54\,\Psi^2\,\omega^2\,p^2\,\omega^{a\,b}\,p'_{;a}+48\,\Psi^3\,\omega^2\,p^2\,\Psi_{;a}\,\omega^{a\,b}$$
$$+36\,\Psi\,\omega^2\,p^3\,\Psi_{;a}\,\omega^{a\,b}+24\,\Psi\,\omega^2\,p^2\,\Psi_{;a}\,\omega^{a\,b}-6\,\Psi^4\,\omega^2\,p''\,PU_{;a}\,\omega^{a\,b}$$
$$-6\,PU\,\Psi^4\,\omega^2\,\omega^{a\,b}\,p''_{;a}+24\,\Psi^4\,\omega^2\,p'\,\omega^{a\,b}\,p'_{;a}+24\,\Psi^2\,\omega\,p^2\,\omega_{;a}\,\omega^{a\,b}$$
$$-4\,\Psi^2\,p'\,\theta^2\,\omega^{a\,b}\,p'_{;a}-6\,\Psi\,\mu\,p^2\,\Psi_{;a}\,\omega^{a\,b}-18\,\Psi^2\,p\,p'\,\omega^{a\,b}\,p'_{;a}$$
$$+4\,\Psi^4\,\omega\,p'\,\omega_{;a}\,\omega^{a\,b}+8\,\Psi^3\,\omega^2\,p'\,\Psi_{;a}\,\omega^{a\,b}-72\,p^3\,\theta^2\,\omega^{a\,b}\,p'_{;a}$$
$$-3\,\Psi^2\,p^2\,\mu_{;a}\,\omega^{a\,b}-9\,\Psi^2\,p^2\,\omega^{a\,b}\,p_{;a}+2\,\Psi^4\,\omega^2\,\omega^{a\,b}\,p'_{;a}-36\,p^4\,\theta\,\omega^{a\,b}\,\theta_{;a}$$
$$-12\,PU\,\Psi^4\,\omega\,p''\,\omega_{;a}\,\omega^{a\,b}-24\,PU\,\Psi^3\,\omega^2\,p''\,\Psi_{;a}\,\omega^{a\,b}=0 \qquad \textbf{(1.5)}$$

Now we use the following identities:

```
> temp4 := `p'`[-A] = `p''`·PU·u[-a]·θ − `p''`·PU·du[-a] / `p'` : T(%);
```

$$p'_{;a}=p''\,PU\,u_a\,\theta-\frac{p''\,PU\,du_a}{p'} \qquad \textbf{(1.6)}$$

```
> temp5 := TEDS(du[-a]·omega[a, b] = 0, TEDS(omega[a, b]·u[-a] = 0, expand(omega[a,
      b]·temp4))) : T(%);
```

$$\omega^{a\,b}\,p'_{;a}=0 \qquad \textbf{(1.7)}$$

```
> temp6 := subs(B = A, eq[64]) : T(%);
```

$$\omega^{a\,b}\,\Psi_{;a}=0 \qquad \textbf{(1.8)}$$

CHECK THIS:

```
> temp7 := `p''`[-A] = `p'''`* PU* theta * u[-a] - `p'''`* PU/ `p'`* du[-a] : T(%);
```

$$p''_{;a}=p'''\,PU\,\theta\,u_a-\frac{p'''\,PU\,du_a}{p'} \qquad \textbf{(1.9)}$$

and so

```
> temp8 := TEDS(du[-a]·omega[a, b] = 0, TEDS(omega[a, b]·u[-a] = 0, expand(omega[a,
      b]·temp7))) : T(%);
```

$$\omega^{a\,b}\,p''_{;a}=0 \qquad \textbf{(1.10)}$$

```
>
```

also

```
> temp9 := subs(a =-a, A =-A, mu[A] = PU* theta * u[a]-PU* du[a]/ `p'`) : T(%);
```

$$\mu_{;a}=PU\,\theta\,u_a-\frac{PU\,du_a}{p'} \qquad \textbf{(1.11)}$$

```
> temp10 := `p`[-A] = `p'`·mu[-A] : T(%);
```

$$p_{;a} = p' \mu_{;a} \quad (1.12)$$

```
> temp11 := expand(TEDS(temp9, expand(MTELS([temp9, temp10], cod(PU=p + mu,
    -a))))) : T(%);
```

$$PU_{;a} = PU\,\theta\, u_a\, p' + PU\,\theta\, u_a - PU\, du_a - \frac{PU\, du_a}{p'} \quad (1.13)$$

hence

```
> temp12 := TEDS(du[-a]·omega[a, b] = 0, TEDS(omega[a, b]·u[-a] = 0, expand(omega[a,
    b]·temp11))) : T(%);
```

$$\omega^{a\ b} PU_{;a} = 0 \quad (1.14)$$

```
>
```

and also

```
> temp13 := TEDS(omega[a, b]·omega[-a] = 0, TEDS(omega[a, b]·u[-a] = 0,
    expand(omega[a, b]·eq[66]))) : T(%);
```

$$\omega^{a\ b} \theta_{;a} = 0 \quad (1.15)$$

```
> temp14 := TEDS(omega[a, b]·du[-a] = 0, TEDS(omega[a, b]·u[-a] = 0, expand(omega[a,
    b]·temp9))) : T(%);
```

$$\omega^{a\ b} \mu_{;a} = 0 \quad (1.16)$$

and so we subs these (temps 5,6,8,9,10,12,13,14) all into the original equation, plus other identities:

```
>
```

```
> temp15 := expand(TEDS(omega[a]·omega[-a] = ω^2, expand(TEDS(du[-a]·omega[a, b]
    = 0, expand(TEDS(omega[a, b]·u[-a] = 0, expand(TEDS(temp14,
    expand(TEDS(temp13, expand(TEDS(temp12, expand(TEDS(temp10,
    expand(TEDS(temp9, expand(TEDS(temp8, expand(TEDS(temp6, expand(TEDS(temp5,
    temp3)))))))))))))))))))))) : T(%);
```

$$-12\, PU\, \Psi^4\, \omega\, p''\, \omega_{;a}\, \omega^{a\ b} + 24\, \Psi^4\, \omega\, p^2\, \omega_{;a}\, \omega^{a\ b} + 4\, \Psi^4\, \omega\, p'\, \omega_{;a}\, \omega^{a\ b} + 36\, \Psi^2\, \omega\, p^3\, \omega_{;a}\, \omega^{a\ b} + 24\, \Psi^2\, \omega\, p^2\, \omega_{;a}\, \omega^{a\ b} = 0 \quad (1.17)$$

```
> temp16 := collect(temp15, [omega[a, b], omega[-A], omega, Ψ^2]) : T(%);
```

$$\left(\left(-12\, PU\, p'' + 24\, p^2 + 4\, p'\right) \Psi^4 + \left(36\, p^3 + 24\, p^2\right) \Psi^2\right) \omega\, \omega_{;a}\, \omega^{a\ b} = 0 \quad (1.18)$$

```
>
```

```
> convert(temp16, string);
```

"((-12*PU*`p"`+24*`p'`^2+4*`p'`)*Psi^4+(36*`p'`^3+24*`p'`^2)*Psi^2)*omega*omega[-A]*omega[a,b] = 0" (1.19)

```
>
```