

> restart;with(Riemann):with(TensorPack): with(Canon):CDF(0): CDS(index):

Chapter XX Tensor analysis using indices - Senovilla et al. - Shearfree for acceleration parallel to vorticity if  $\sigma_{ab}=0 \Rightarrow \omega\Theta=0$

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eq72a - time differentiation of SSSeq72

$$\begin{aligned} &> \text{read "EFE": read "SFE": read "fids": read "Seneqs80":} \\ &> \text{SSSeq72 := } ((3 * p'^2 / \Psi^2 + 1/3) * \theta^2 - 2 * (\Psi^2 + 1) * \omega^2 + 1/2 * \mu + 3/2 * p) * p' / \Psi^2 = \left( 3 \cdot \left( \frac{p'}{\Psi} \right)^2 + \frac{1}{3} - P \cdot p'' / p' \right) * \omega^2 : T(\%); \\ &\frac{\left( \left( \frac{3p^2}{\Psi^2} + \frac{1}{3} \right) \theta^2 - 2 (\Psi^2 + 1) \omega^2 + \frac{1}{2} \mu + \frac{3}{2} p \right) p'}{\Psi^2} = \left( \frac{3p^2}{\Psi^2} + \frac{1}{3} - \frac{P \cdot p''}{p'} \right) \omega^2 \quad (1.1) \end{aligned}$$

proof of eq72a: We commence with SSSeq72:

> temp := eq[72]: T(%);

$$\frac{\left( \left( \frac{3p^2}{\Psi^2} + \frac{1}{3} \right) \theta^2 - 2 (\Psi^2 + 1) \omega^2 + \frac{1}{2} \mu + \frac{3}{2} p \right) p'}{\Psi^2} = \left( \frac{3p^2}{\Psi^2} + \frac{1}{3} - \frac{P \cdot p''}{p'} \right) \omega^2 \quad (1.2)$$

$$\begin{aligned} &> \text{temp1 := expand}(6 \cdot \Psi^4 \cdot p' \cdot \text{expand}(rhs(temp) - lhs(temp)) = 0) : T(\%); \\ &- 6 P \cdot U \Psi^4 \omega^2 p'' + 12 \Psi^4 \omega^2 p^2 + 2 \Psi^4 \omega^2 p' + 18 \Psi^2 \omega^2 p^3 + 12 \Psi^2 \omega^2 p^2 - 2 \Psi^2 p^2 \theta^2 \\ &\quad - 18 p^4 \theta^2 - 3 \Psi^2 \mu p^2 - 9 \Psi^2 p p^2 = 0 \quad (1.3) \end{aligned}$$

taking the time derivative:

> temp2 := dotT(temp1): T(%);

$$\begin{aligned} &- 6 P \cdot U \Psi^4 \cdot \text{dotp}'' \omega^2 - 12 P \cdot U \Psi^4 \omega p'' \cdot \text{dotomega} - 24 P \cdot U \Psi^3 \cdot \text{dotPsi} \omega^2 p'' \\ &\quad - 6 \Psi^4 \cdot \text{dotPU} \omega^2 p'' + 24 \Psi^4 \cdot \text{dotp}' \omega^2 p' + 24 \Psi^4 \omega p^2 \cdot \text{dotomega} + 48 \Psi^3 \cdot \text{dotPsi} \omega^2 p^2 \\ &\quad + 2 \Psi^4 \cdot \text{dotp}' \omega^2 + 4 \Psi^4 \omega p' \cdot \text{dotomega} + 8 \Psi^3 \cdot \text{dotPsi} \omega^2 p' + 54 \Psi^2 \cdot \text{dotp}' \omega^2 p^2 \\ &\quad + 36 \Psi^2 \omega p^3 \cdot \text{dotomega} + 36 \Psi \cdot \text{dotPsi} \omega^2 p^3 + 24 \Psi^2 \cdot \text{dotp}' \omega^2 p' - 4 \Psi^2 \cdot \text{dotp}' p' \theta^2 \\ &\quad + 24 \Psi^2 \omega p^2 \cdot \text{dotomega} - 4 \Psi^2 p^2 \theta \cdot \text{dottheta} + 24 \Psi \cdot \text{dotPsi} \omega^2 p^2 - 4 \Psi \cdot \text{dotPsi} p^2 \theta^2 \\ &\quad - 72 \cdot \text{dotp}' p^3 \theta^2 - 36 p^4 \theta \cdot \text{dottheta} - 3 \Psi^2 \cdot \text{dotmu} p^2 - 9 \Psi^2 \cdot \text{dotp} p^2 - 6 \Psi^2 \cdot \text{dotp}' \mu p' \\ &\quad - 18 \Psi^2 \cdot \text{dotp}' p p' - 6 \Psi \cdot \text{dotPsi} \mu p^2 - 18 \Psi \cdot \text{dotPsi} p p^2 = 0 \quad (1.4) \end{aligned}$$

>

Now we use the following identities:

$$> temp3 := dotomega = \theta \cdot \omega \cdot p' - \frac{2}{3} \cdot \theta \cdot \omega : T(\%);$$

$$dotomega = \theta \omega p' - \frac{2}{3} \theta \omega \quad (1.5)$$

$$> temp4 := `dotp` = - p'' \cdot \theta \cdot PU : T(\%);$$

$$dotp' = - p'' \theta PU \quad (1.6)$$

$$> temp5 := eq[65] : T(\%);$$

$$dotPsi = \left( -\frac{p'' \mu}{p'} - \frac{p'' p}{p'} + \frac{3 p^2}{\Psi^2} + \frac{1}{3} \right) \Psi \theta \quad (1.7)$$

$$> temp6 := `dotp`` = - p''' \cdot \theta \cdot PU : T(\%);$$

$$dotp'' = - p''' \theta PU \quad (1.8)$$

$$> temp7 := dotPU = - (1 + p') \cdot PU \cdot \theta : T(\%);$$

$$dotPU = - (1 + p') PU \theta \quad (1.9)$$

$$> #temp8 := isolate(eq[20], dottheta) : T(\%);$$

$$> temp8 := subs(dottheta = dottheta, eq[69]) : T(\%);$$

$$dottheta = \frac{3 p^2 \theta^2}{\Psi^2} \quad (1.10)$$

$$> temp9 := TEDS(mu + p = PU, isolate(eq[30], dotmu)) : T(\%);$$

$$dotmu = - \theta PU \quad (1.11)$$

$$> temp10 := `dotp` = - p' \cdot \theta \cdot PU : T(\%);$$

$$dotp = - p' \theta PU \quad (1.12)$$

$$> temp11 := du[a, -A] = 2 \cdot \Psi^2 \cdot \omega^2 + Psi[-A] \cdot omega[a] : T(\%);$$

$$du^a_{,a} = 2 \Psi^2 \omega^2 + \Psi_{,a} \omega^a \quad (1.13)$$

$$> temp12 := eq[71] : T(\%);$$

$$\Psi_{,a} \omega^a = \left( \frac{3 p^2}{\Psi^2} + \frac{1}{3} \right) \theta^2 - 2 (\Psi^2 + 1) \omega^2 + \frac{1}{2} \mu + \frac{3}{2} p \quad (1.14)$$

Now substituting all of these:

$$> temp13 := expand \left( \frac{p' \cdot \Psi^2 \cdot \omega^2}{\theta \cdot p'} TEDS(temp12, TEDS(PU=p+mu, TEDS(temp11, expand(TEDS(temp10, expand(TEDS(temp9, expand(TEDS(temp8, expand(TEDS(temp7, expand(TEDS(temp6, expand(TEDS(temp5, TEDS(temp4, expand(TEDS(temp3, temp2))))))))))))))))))) : T(\%) \right.$$

$$12 \omega^4 \Psi^6 \mu p p''' + \frac{48 \omega^4 \Psi^6 \mu p'^2 p}{p'} + 8 \omega^2 p' \Psi^4 \mu p'' \theta^2 + 8 \omega^2 p' \Psi^4 p p'' \theta^2 \quad (1.15)$$

$$\begin{aligned}
& + 72 \omega^2 p^3 \theta^2 p'' \Psi^2 \mu + 72 \omega^2 p^3 \theta^2 p'' \Psi^2 p + 48 \omega^2 p' \Psi^4 \mu p p'' + \frac{24 \omega^4 \Psi^6 \mu^2 p'^2}{p'} \\
& - 78 \omega^4 p' \Psi^6 \mu p'' + 6 \omega^4 \Psi^6 p^2 p''' + \frac{24 \omega^4 \Psi^6 p^2 p'^2}{p'} - 78 \omega^4 p' \Psi^6 p p'' \\
& - 4 \omega^4 \Psi^6 \mu p'' - 4 \omega^4 \Psi^6 p p'' - 162 \omega^4 p^2 \Psi^4 \mu p'' - 162 \omega^4 p^2 \Psi^4 p p'' \\
& - 48 \omega^4 p' \Psi^4 \mu p'' - 48 \omega^4 p' \Psi^4 p p'' + 12 \omega^2 p' \Psi^4 \mu^2 p'' + 36 \omega^2 p' \Psi^4 p^2 p'' \\
& + 6 \omega^4 \Psi^6 \mu^2 p''' + 24 \omega^4 p^3 \Psi^6 + 4 \omega^4 p^2 \Psi^6 + 180 \omega^4 p^4 \Psi^4 + 36 \omega^4 p^3 \Psi^4 \\
& + 108 \omega^4 p^5 \Psi^2 - 8 \omega^4 p^2 \Psi^4 + 72 \omega^4 p^4 \Psi^2 - 108 \omega^2 p^6 \theta^2 + 9 \omega^2 p^3 \Psi^4 \mu \\
& + 9 \omega^2 p^3 \Psi^4 p - \frac{4}{3} \omega^2 p^2 \Psi^4 \theta^2 - 24 \omega^2 p^4 \theta^2 \Psi^2 + \omega^2 p^2 \Psi^4 \mu - 3 \omega^2 p^2 \Psi^4 p \\
& - 18 \omega^2 p^4 \mu \Psi^2 - 54 \omega^2 p^4 p \Psi^2 = 0
\end{aligned}$$

> *temp14* := isolate(*temp13*, *p'''*) : T(%);

$$\begin{aligned}
p''' &= \frac{1}{6 \Psi^6 \mu^2 \omega^4 + 12 \Psi^6 \mu \omega^4 p + 6 \Psi^6 \omega^4 p^2} \left( -\frac{48 \omega^4 \Psi^6 \mu p'^2 p}{p'} - 8 \omega^2 p' \Psi^4 \mu p'' \theta^2 \right. & (1.16) \\
& - 8 \omega^2 p' \Psi^4 p p'' \theta^2 - 72 \omega^2 p^3 \theta^2 p'' \Psi^2 \mu - 72 \omega^2 p^3 \theta^2 p'' \Psi^2 p - 48 \omega^2 p' \Psi^4 \mu p p'' \\
& - \frac{24 \omega^4 \Psi^6 \mu^2 p'^2}{p'} + 78 \omega^4 p' \Psi^6 \mu p'' - \frac{24 \omega^4 \Psi^6 p^2 p'^2}{p'} + 78 \omega^4 p' \Psi^6 p p'' \\
& + 4 \omega^4 \Psi^6 \mu p'' + 4 \omega^4 \Psi^6 p p'' + 162 \omega^4 p^2 \Psi^4 \mu p'' + 162 \omega^4 p^2 \Psi^4 p p'' \\
& + 48 \omega^4 p' \Psi^4 \mu p'' + 48 \omega^4 p' \Psi^4 p p'' - 12 \omega^2 p' \Psi^4 \mu^2 p'' - 36 \omega^2 p' \Psi^4 p^2 p'' \\
& - 24 \omega^4 p^3 \Psi^6 - 4 \omega^4 p^2 \Psi^6 - 180 \omega^4 p^4 \Psi^4 - 36 \omega^4 p^3 \Psi^4 - 108 \omega^4 p^5 \Psi^2 \\
& + 8 \omega^4 p^2 \Psi^4 - 72 \omega^4 p^4 \Psi^2 + 108 \omega^2 p^6 \theta^2 - 9 \omega^2 p^3 \Psi^4 \mu - 9 \omega^2 p^3 \Psi^4 p \\
& + \frac{4}{3} \omega^2 p^2 \Psi^4 \theta^2 + 24 \omega^2 p^4 \theta^2 \Psi^2 - \omega^2 p^2 \Psi^4 \mu + 3 \omega^2 p^2 \Psi^4 p + 18 \omega^2 p^4 \mu \Psi^2 \\
& \left. + 54 \omega^2 p^4 p \Psi^2 \right)
\end{aligned}$$

> convert(*temp13*, string);

$$\begin{aligned}
& "12*\omega^4*\Psii^6*\mu*p*'p''' + 48*\omega^4/\p**\Psii^6*\mu*'p''^2*p + 8*\omega^2*\p** \\
& \Psii^4*\mu*'p**\theta^2 + 8*\omega^2*\p**\Psii^4*p*'p**\theta^2 + 72*\omega^2*\p'^3*
\end{aligned} \quad (1.17)$$

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theta^2*p''*Psi^2*mu+72*omega^2*p''^3*theta^2*p''*Psi^2*p+48*omega^2*p''*
Psi^4*mu*p*p''+24*omega^4/p''*Psi^6*mu^2*p''^2-78*omega^4*p''*Psi^6*mu*
`p''+6*omega^4*Psi^6*p^2*p'''+24*omega^4/p''*Psi^6*p^2*p''^2-78*omega^4*
`p''*Psi^6*p*p''-4*omega^4*Psi^6*mu*p''-4*omega^4*Psi^6*p*p''-162*omega^4*
`p''^2*Psi^4*mu*p''-162*omega^4*p''^2*Psi^4*p*p''-48*omega^4*p''*Psi^4*mu*
`p''-48*omega^4*p''*Psi^4*p*p''+12*omega^2*p''*Psi^4*mu^2*p''+36*
omega^2*p''*Psi^4*p^2*p''+6*omega^4*Psi^6*mu^2*p'''+24*omega^4*p''^3*
Psi^6+4*omega^4*p''^2*Psi^6+180*omega^4*p''^4*Psi^4+36*omega^4*p''^3*
Psi^4+108*omega^4*p''^5*Psi^2-8*omega^4*p''^2*Psi^4+72*omega^4*p''^4*
Psi^2-108*omega^2*p''^6*theta^2+9*omega^2*p''^3*Psi^4*mu+9*omega^2*
`p''^3*Psi^4*p-4/3*omega^2*p''^2*Psi^4*theta^2-24*omega^2*p''^4*theta^2*
Psi^2+omega^2*p''^2*Psi^4*mu-3*omega^2*p''^2*Psi^4*p-18*omega^2*p''^4*
mu*Psi^2-54*omega^2*p''^4*p*Psi^2 = 0"

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$$> temp4 := `dotp'=-`p''\cdot\theta\cdot PU : T(\%); \quad (1.18)$$

$$dotp' = -p''\theta PU$$

$$> temp5 := eq[65] : T(\%); \quad (1.19)$$

$$dotPsi = \left( -\frac{p''\mu}{p'} - \frac{p''p}{p'} + \frac{3p^2}{\Psi^2} + \frac{1}{3} \right) \Psi \theta$$

$$> temp6 := `dotp''=-`p'''\cdot theta\cdot PU: T(\%); \quad (1.20)$$

$$dotp'' = -p'''\theta PU$$

$$> temp7 := dotPU=- (1 + `p')\cdot PU\cdot theta : T(\%); \quad (1.21)$$

$$dotPU = -(1 + p') PU\theta$$

$$> #temp8:=isolate(eq[20], dottheta) : T(\%);$$

$$> temp8 := subs(thetaDot=dottheta, eq[69]) : T(\%); \quad (1.22)$$

$$dottheta = \frac{3p^2\theta^2}{\Psi^2}$$

$$> temp9 := TEDS(mu + p = PU, isolate(eq[30], dotmu)) : T(\%); \quad (1.23)$$

$$dotmu = -\theta PU$$

$$> temp10 := `dotp`=-`p''\cdot\theta\cdot PU : T(\%); \quad (1.24)$$

$$dotp = -p'\theta PU$$

$$> temp11 := du[a, -A] = 2\cdot\Psi^2\cdot\omega^2 + Psi[-A]\cdot omega[a] : T(\%); \quad (1.25)$$

$$du^a_{,a} = 2\Psi^2\omega^2 + \Psi_{,a}\omega^a$$

$$> temp12 := eq[71] : T(\%);$$

$$\Psi_{,a} \omega^a = \left( \frac{3p^2}{\Psi^2} + \frac{1}{3} \right) \theta^2 - 2 (\Psi^2 + 1) \omega^2 + \frac{1}{2} \mu + \frac{3}{2} p \quad (1.26)$$

Now substituting all of these:

$$\begin{aligned} > temp13 := expand\left(\frac{\dot{p}' \cdot \Psi^2 \cdot \omega^2}{\text{theta} \cdot \dot{p}'} TEDS(temp12, TEDS(PU=p+\mu, TEDS(temp11, \\ expand(TEDS(temp10, expand(TEDS(temp9, expand(TEDS(temp8, \\ expand(TEDS(temp7, expand(TEDS(temp6, expand(TEDS(temp5, TEDS(temp4, \\ expand(TEDS(temp3, temp2))))))))))))))) : T(\%) \right. \\ & 12 \omega^4 \Psi^6 \mu p p''' + \frac{48 \omega^4 \Psi^6 \mu p'^2 p}{p'} + 8 \omega^2 p' \Psi^4 \mu p'' \theta^2 + 8 \omega^2 p' \Psi^4 p p'' \theta^2 \\ & + 72 \omega^2 p^3 \theta^2 p'' \Psi^2 \mu + 72 \omega^2 p^3 \theta^2 p'' \Psi^2 p + 48 \omega^2 p' \Psi^4 \mu p p'' + \frac{24 \omega^4 \Psi^6 \mu^2 p'^2}{p'} \\ & - 78 \omega^4 p' \Psi^6 \mu p'' + 6 \omega^4 \Psi^6 p^2 p''' + \frac{24 \omega^4 \Psi^6 p^2 p'^2}{p'} - 78 \omega^4 p' \Psi^6 p p'' \\ & - 4 \omega^4 \Psi^6 \mu p'' - 4 \omega^4 \Psi^6 p p'' - 162 \omega^4 p^2 \Psi^4 \mu p'' - 162 \omega^4 p^2 \Psi^4 p p'' \\ & - 48 \omega^4 p' \Psi^4 \mu p'' - 48 \omega^4 p' \Psi^4 p p'' + 12 \omega^2 p' \Psi^4 \mu^2 p'' + 36 \omega^2 p' \Psi^4 p^2 p'' \\ & + 6 \omega^4 \Psi^6 \mu^2 p''' + 24 \omega^4 p^3 \Psi^6 + 4 \omega^4 p^2 \Psi^6 + 180 \omega^4 p^4 \Psi^4 + 36 \omega^4 p^3 \Psi^4 \\ & + 108 \omega^4 p^5 \Psi^2 - 8 \omega^4 p^2 \Psi^4 + 72 \omega^4 p^4 \Psi^2 - 108 \omega^2 p^6 \theta^2 + 9 \omega^2 p^3 \Psi^4 \mu \\ & + 9 \omega^2 p^3 \Psi^4 p - \frac{4}{3} \omega^2 p^2 \Psi^4 \theta^2 - 24 \omega^2 p^4 \theta^2 \Psi^2 + \omega^2 p^2 \Psi^4 \mu - 3 \omega^2 p^2 \Psi^4 p \\ & - 18 \omega^2 p^4 \mu \Psi^2 - 54 \omega^2 p^4 p \Psi^2 = 0 \end{aligned} \quad (1.27)$$

$$> temp14 := isolate(temp13, \dot{p}''') : T(\%);$$

$$\begin{aligned} \dot{p}''' = & \frac{1}{6 \Psi^6 \mu^2 \omega^4 + 12 \Psi^6 \mu \omega^4 p + 6 \Psi^6 \omega^4 p^2} \left( - \frac{48 \omega^4 \Psi^6 \mu p'^2 p}{p'} - 8 \omega^2 p' \Psi^4 \mu p'' \theta^2 \right. \\ & - 8 \omega^2 p' \Psi^4 p p'' \theta^2 - 72 \omega^2 p^3 \theta^2 p'' \Psi^2 \mu - 72 \omega^2 p^3 \theta^2 p'' \Psi^2 p - 48 \omega^2 p' \Psi^4 \mu p p'' \\ & - \frac{24 \omega^4 \Psi^6 \mu^2 p'^2}{p'} + 78 \omega^4 p' \Psi^6 \mu p'' - \frac{24 \omega^4 \Psi^6 p^2 p'^2}{p'} + 78 \omega^4 p' \Psi^6 p p'' \\ & + 4 \omega^4 \Psi^6 \mu p'' + 4 \omega^4 \Psi^6 p p'' + 162 \omega^4 p^2 \Psi^4 \mu p'' + 162 \omega^4 p^2 \Psi^4 p p'' \\ & + 48 \omega^4 p' \Psi^4 \mu p'' + 48 \omega^4 p' \Psi^4 p p'' - 12 \omega^2 p' \Psi^4 \mu^2 p'' - 36 \omega^2 p' \Psi^4 p^2 p'' \\ & - 24 \omega^4 p^3 \Psi^6 - 4 \omega^4 p^2 \Psi^6 - 180 \omega^4 p^4 \Psi^4 - 36 \omega^4 p^3 \Psi^4 - 108 \omega^4 p^5 \Psi^2 \end{aligned} \quad (1.28)$$

$$\begin{aligned}
& + 8 \omega^4 p^2 \Psi^4 - 72 \omega^4 p^4 \Psi^2 + 108 \omega^2 p^6 \theta^2 - 9 \omega^2 p^3 \Psi^4 \mu - 9 \omega^2 p^3 \Psi^4 p \\
& + \frac{4}{3} \omega^2 p^2 \Psi^4 \theta^2 + 24 \omega^2 p^4 \theta^2 \Psi^2 - \omega^2 p^2 \Psi^4 \mu + 3 \omega^2 p^2 \Psi^4 p + 18 \omega^2 p^4 \mu \Psi^2 \\
& + 54 \omega^2 p^4 p \Psi^2
\end{aligned}$$

> *convert(temp13, string);*

$$"12*\omega^4*\Psii^6*\mu*\text{p}^*\text{p}''' + 48*\omega^4/\text{p}''*\Psii^6*\mu*\text{p}'''^2\text{p} + 8*\omega^2*\text{p}''^* \quad (1.29)$$

$$\begin{aligned}
& \Psii^4*\mu*\text{p}'''^2 + 8*\omega^2*\text{p}''^*\Psii^4*\text{p}''*\theta^2 + 72*\omega^2*\text{p}'''^3 \\
& \theta^2 + 2*\text{p}'''^2\Psii^2*\mu + 72*\omega^2*\text{p}'''^3\theta^2 + 2*\text{p}'''^2\Psii^2*\text{p} + 48*\omega^2*\text{p}''^* \\
& \Psii^4*\mu*\text{p}'' + 24*\omega^4/\text{p}''*\Psii^6*\mu^2*\text{p}'''^2 - 78*\omega^4*\text{p}''^*\Psii^6*\mu* \\
& \text{p}''' + 6*\omega^4\Psii^6*\text{p}^2*\text{p}''' + 24*\omega^4/\text{p}''*\Psii^6*\text{p}^2*\text{p}'''^2 - 78*\omega^4*\text{p}''^* \\
& \Psii^6*\text{p}^*\text{p}''' - 4*\omega^4\Psii^6*\mu*\text{p}''' - 4*\omega^4\Psii^6*\text{p}^*\text{p}''' - 162*\omega^4*\text{p}''^* \\
& \text{p}'''^2\Psii^4*\mu*\text{p}''' - 162*\omega^4*\text{p}'''^2\Psii^4*\text{p}^*\text{p}''' - 48*\omega^4*\text{p}''^*\Psii^4*\mu* \\
& \text{p}''' - 48*\omega^4*\text{p}''^*\Psii^4*\text{p}^*\text{p}''' + 12*\omega^4*\text{p}'''^2\Psii^4*\mu^2*\text{p}''' + 36* \\
& \omega^2*\text{p}''^*\Psii^4*\text{p}^2*\text{p}''' + 6*\omega^4\Psii^6*\mu^2*\text{p}''' + 24*\omega^4*\text{p}'''^3* \\
& \Psii^6 + 4*\omega^4*\text{p}'''^2\Psii^6 + 180*\omega^4/\text{p}''^4\Psii^4 + 36*\omega^4*\text{p}'''^3* \\
& \Psii^4 + 108*\omega^4*\text{p}'''^5\Psii^2 - 8*\omega^4*\text{p}'''^2\Psii^4 + 72*\omega^4*\text{p}'''^4* \\
& \Psii^2 - 108*\omega^4*\text{p}'''^6\theta^2 + 9*\omega^4*\text{p}'''^3\Psii^4*\mu + 9*\omega^4*\text{p}'''^3* \\
& \text{p}'''^3\Psii^4*\text{p} - 4/3*\omega^4*\text{p}'''^2\Psii^4*\theta^2 - 24*\omega^4*\text{p}'''^4*\theta^2* \\
& \Psii^2 + \omega^4*\text{p}'''^2\Psii^4*\mu - 3*\omega^4*\text{p}'''^2\Psii^4*\text{p} - 18*\omega^4*\text{p}'''^4* \\
& \mu*\Psii^2 - 54*\omega^4*\text{p}'''^4*\text{p}*\Psii^2 = 0"
\end{aligned}$$

> *convert(temp14, string);*

$$\text{p}''' = (-48*\omega^4/\text{p}''*\Psii^6*\mu*\text{p}'''^2\text{p} - 8*\omega^4*\text{p}''*\Psii^4*\mu*\text{p}'''^2\theta^2 - 8* \quad (1.30)$$

$$\begin{aligned}
& \omega^2*\text{p}''*\Psii^4*\text{p}^*\text{p}'''^2\theta^2 - 72*\omega^2*\text{p}'''^3\theta^2 + 2*\text{p}'''^2\Psii^2*\mu - 72* \\
& \omega^2*\text{p}'''^3\theta^2 + 2*\text{p}'''^2\Psii^2*\text{p} - 48*\omega^2*\text{p}''*\Psii^4*\mu*\text{p}''' - 24* \\
& \omega^4/\text{p}''*\Psii^6*\mu^2*\text{p}'''^2 + 78*\omega^4*\text{p}''^*\Psii^6*\mu*\text{p}''' - 24*\omega^4*\text{p}''^* \\
& \Psii^6*\text{p}^2*\text{p}'''^2 + 78*\omega^4*\text{p}''^*\Psii^6*\text{p}^*\text{p}''' + 4*\omega^4\Psii^6*\mu*\text{p}''' + 4* \\
& \omega^4\Psii^6*\text{p}^*\text{p}''' + 162*\omega^4*\text{p}'''^2\Psii^4*\mu*\text{p}''' + 162*\omega^4*\text{p}'''^2* \\
& \Psii^4*\text{p}^*\text{p}''' + 48*\omega^4*\text{p}''*\Psii^4*\mu*\text{p}''' + 48*\omega^4*\text{p}''*\Psii^4*\text{p}^*\text{p}''' - 12* \\
& \omega^2*\text{p}''*\Psii^4*\mu^2*\text{p}''' - 36*\omega^2*\text{p}''*\Psii^4*\text{p}^2*\text{p}''' - 24*\omega^4*\text{p}'''^3* \\
& \Psii^6 - 4*\omega^4*\text{p}'''^2\Psii^6 - 180*\omega^4*\text{p}'''^4\Psii^4 - 36*\omega^4*\text{p}'''^3\Psii^4 \\
& - 108*\omega^4*\text{p}'''^5\Psii^2 + 8*\omega^4*\text{p}'''^2\Psii^4*\text{p} - 72*\omega^4*\text{p}'''^4* \\
& \Psii^2 + 108*\omega^4*\text{p}'''^6\theta^2 - 9*\omega^4*\text{p}'''^3\Psii^4*\mu - 9*\omega^4*\text{p}'''^4* \\
& \Psii^4*\text{p} + 4/3*\omega^4*\text{p}'''^2\Psii^4*\theta^2 + 24*\omega^4*\text{p}'''^4*\theta^2*\Psii^2 -
\end{aligned}$$

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omega^2*p''^2*Psi^4*mu+3*omega^2*p''^2*Psi^4*p+18*omega^2*p''^4*mu*
Psi^2+54*omega^2*p''^4*p*Psi^2)/(6*Psi^6*mu^2*omega^4+12*Psi^6*mu*
omega^4*p+6*Psi^6*omega^4*p^2)"

```

>  $\text{temp15} := \text{collect}(\text{temp13}, [\text{p}''', \text{Psi}, \text{omega}]) : T(\%)$ ;

$$\begin{aligned}
& (6\mu^2 + 12\mu p + 6p^2) \omega^4 \Psi^6 p''' + \left( \frac{48\mu p'^2 p}{p'} + \frac{24\mu^2 p'^2}{p'} - 78p'\mu p'' + \frac{24p^2 p'^2}{p'} \right. \\
& \left. - 78p'p p'' - 4p''\mu - 4p''p + 24p^3 + 4p^2 \right) \omega^4 \Psi^6 + \left( (-162\mu p^2 p'' \right. \\
& \left. - 162p p^2 p'' + 180p^4 - 48\mu p' p'' - 48p p' p'' + 36p^3 - 8p^2) \omega^4 + \left( 8p'\mu p''\theta^2 \right. \right. \\
& \left. + 8p'p p''\theta^2 + 48p'\mu p p'' + 12p'\mu^2 p'' + 36p'p^2 p'' + 9p^3\mu + 9p^3p - \frac{4}{3}p^2\theta^2 \right. \\
& \left. + p^2\mu - 3p^2p \right) \omega^2 \right) \Psi^4 + \left( (108p^5 + 72p^4) \omega^4 + (72\mu p^3 p''\theta^2 + 72p p^3 p''\theta^2 \right. \\
& \left. - 24p^4\theta^2 - 18\mu p^4 - 54p p^4) \omega^2 \right) \Psi^2 - 108\omega^2 p^6 \theta^2 = 0
\end{aligned} \tag{1.31}$$

>  $\text{temp16} := \text{isolate}(\text{temp15}, \text{op}(1, \text{op}(1, \text{temp15}))) : T(\%)$ ;

$$\begin{aligned}
& (6\mu^2 + 12\mu p + 6p^2) \omega^4 \Psi^6 p''' = - \left( \frac{48\mu p'^2 p}{p'} + \frac{24\mu^2 p'^2}{p'} - 78p'\mu p'' + \frac{24p^2 p'^2}{p'} \right. \\
& \left. - 78p'p p'' - 4p''\mu - 4p''p + 24p^3 + 4p^2 \right) \omega^4 \Psi^6 - \left( (-162\mu p^2 p'' \right. \\
& \left. - 162p p^2 p'' + 180p^4 - 48\mu p' p'' - 48p p' p'' + 36p^3 - 8p^2) \omega^4 + \left( 8p'\mu p''\theta^2 \right. \right. \\
& \left. + 8p'p p''\theta^2 + 48p'\mu p p'' + 12p'\mu^2 p'' + 36p'p^2 p'' + 9p^3\mu + 9p^3p - \frac{4}{3}p^2\theta^2 \right. \\
& \left. + p^2\mu - 3p^2p \right) \omega^2 \right) \Psi^4 - \left( (108p^5 + 72p^4) \omega^4 + (72\mu p^3 p''\theta^2 + 72p p^3 p''\theta^2 \right. \\
& \left. - 24p^4\theta^2 - 18\mu p^4 - 54p p^4) \omega^2 \right) \Psi^2 + 108\omega^2 p^6 \theta^2
\end{aligned} \tag{1.32}$$

>  $\text{convert}(\text{temp16}, \text{string})$ ;

$$\begin{aligned}
& "(6*mu^2+12*mu*p+6*p^2)*omega^4*Psi^6*p''' = -(48/p**mu*p''^2*p+24/p** \\
& mu^2*p''^2-78*p**mu*p''+24/p**p^2*p''^2-78*p**p*p''-4*p**mu-4*p** \\
& p+24*p''^3+4*p''^2)*omega^4*Psi^6-((-162*mu*p''^2*p''-162*p*p''^2* \\
& *p''+180*p''^4-48*mu*p**p''-48*p*p**p''+36*p''^3-8*p''^2)*omega^4+(8* \\
& *p**mu*p**theta^2+8*p**p**p**theta^2+48*p**mu*p*p''+12*p**mu^2* \\
& *p''+36*p**p^2*p''+9*p**3*mu+9*p**3*p-4/3*p**2*theta^2+p**2*mu-3* \\
& *p**2*p)*omega^2)*Psi^4-((108*p**5+72*p**4)*omega^4+(72*mu*p**3*p** \\
& theta^2+72*p*p**3*p**theta^2-24*p**4*theta^2-18*mu*p**4-54*p*p**4)*
\end{aligned} \tag{1.33}$$

$$\left[ \begin{array}{l} \text{omega}^2) * \text{Psi}^2 + 108 * \text{omega}^2 * 'p'^6 * \text{theta}^2 \\ \text{v} \end{array} \right]$$