```
> restart;with(Riemann):with(TensorPack): with(Canon):CDF(0): CDS(index):
```

Chapter XX Tensor analysis using indices - Senovilla et al. - Shearfree for acceleration parallel to vorticity if $\sigma_{a\,b}$=0 =>ωΘ=0

Author: Peter Huf

eq70

```
> read "EFE" : read "SFE" :read "fids" :read "Seneqs80" :
> eq[70]
    := parse("1/Psi^2*`p'`*Psi[-A]*omega[a] = 1/3/Psi^2/`p'`*(-3*PU*Psi^2*`p''`+Psi^2*
   `p'`+9*`p'`^3)*omega^2") : T(%);
```

$$\frac{p'\Psi_{;a}\,\omega^{a}}{\Psi^2} = \frac{1}{3}\,\frac{\left(-3\,PU\,\Psi^2\,p''+\Psi^2\,p'+9\,p'^3\right)\omega^2}{\Psi^2\,p'} \quad \textbf{(1.1)}$$

proof of eq70: We commence with eq69

```
> temp := eq[69] : T(%);
```

$$thetadot = \frac{3\,p'^2\,\theta^2}{\Psi^2} \quad \textbf{(1.2)}$$

```
> temp2 := eq[66] : T(%);
```

$$\theta_{;a} = \frac{3\,p'\,\theta\,\omega_{a}}{\Psi} - u_{a}\,thetadot \quad \textbf{(1.3)}$$

```
> temp3 := expand(TEDS(temp, temp2)) : T(%);
```

$$\theta_{;a} = \frac{3\,p'\,\theta\,\omega_{a}}{\Psi} - \frac{3\,u_{a}\,p'^2\,\theta^2}{\Psi^2} \quad \textbf{(1.4)}$$

```
>
```

Now we argue that since:

```
> temp4 := theta[-A,-B] = theta[-B,-A] : T(%);
```

$$\theta_{;a\,;b} = \theta_{;b\,;a} \quad \textbf{(1.5)}$$

so we have

```
> temp5 := expand(cod(temp3,-b)) : T(%);
```

$$\theta_{;a\,;b} = \frac{3\,p'_{;b}\,\theta\,\omega_{a}}{\Psi} + \frac{3\,p'\,\theta_{;b}\,\omega_{a}}{\Psi} + \frac{3\,p'\,\theta\,\omega_{a\,;b}}{\Psi} - \frac{3\,p'\,\theta\,\omega_{a}\,\Psi_{;b}}{\Psi^2} - \frac{3\,u_{a\,;b}\,p'^2\,\theta^2}{\Psi^2} - \frac{6\,u_{a}\,p'\,p'_{;b}\,\theta^2}{\Psi^2} + \frac{6\,u_{a}\,p'^2\,\Psi_{;b}\,\theta^2}{\Psi^3} - \frac{6\,u_{a}\,p'^2\,\theta\,\theta_{;b}}{\Psi^2} \quad \textbf{(1.6)}$$

and also

```
> temp6 := cod(subs(a = b, A = B, temp3),-a) : T(%);
```

$$\theta_{;b;a} = \frac{3\,p'_{;a}\,\theta\,\omega_b}{\Psi} + \frac{3\,p'\,\theta_{;a}\,\omega_b}{\Psi} + \frac{3\,p'\,\theta\,\omega_{b;a}}{\Psi} - \frac{3\,p'\,\theta\,\omega_b\,\Psi_{;a}}{\Psi^2} - \frac{3\,u_{b;a}\,p^2\,\theta^2}{\Psi^2} - \frac{6\,u_b\,p'\,p'_{;a}\,\theta^2}{\Psi^2} + \frac{6\,u_b\,p^2\,\Psi_{;a}\,\theta^2}{\Psi^3} - \frac{6\,u_b\,p^2\,\theta\,\theta_{;a}}{\Psi^2} \tag{1.7}$$

>

Also we note a previous kinematic identity:

> temp7 := parse("dottheta[-A]=theta[-F]*du[f]*u[-a]-theta[-F]*omega[f,-a]-theta*theta[-A]+4*omega*omega[-A]-1/2*mu[-A]-3/2*p[-A]+du[e,-E,-A]+1/9*theta^3*u[-a]-2/3*theta*u[-a]*omega^2+1/6*theta*u[-a]*mu+1/2*theta*u[-a]*p-1/3*theta*u[-a]*du[e,-E]"):T(%);

$$dottheta_{;a} = \theta_{;f}\,du^{f}\,u_a - \theta_{;f}\,\omega^{f}{}_{a} - \theta\,\theta_{;a} + 4\,\omega\,\omega_{;a} - \frac{1}{2}\,\mu_{;a} - \frac{3}{2}\,p_{;a} + du^{e}{}_{;e;a} + \frac{1}{9}\,\theta^3\,u_a - \frac{2}{3}\,\theta\,u_a\,\omega^2 + \frac{1}{6}\,\theta\,u_a\,\mu + \frac{1}{2}\,\theta\,u_a\,p - \frac{1}{3}\,\theta\,u_a\,du^{e}{}_{;e} \tag{1.8}$$

>

is the same as temp5 contracted with u[b]. Let us explore this momentarily:

> temp8 := expand(temp5·u[b]) : T(%);

$$u^b\,\theta_{;a;b} = \frac{3\,u^b\,p'_{;b}\,\theta\,\omega_a}{\Psi} + \frac{3\,u^b\,p'\,\theta_{;b}\,\omega_a}{\Psi} + \frac{3\,u^b\,p'\,\theta\,\omega_{a;b}}{\Psi} - \frac{3\,u^b\,p'\,\theta\,\omega_a\,\Psi_{;b}}{\Psi^2} - \frac{3\,u^b\,u_{a;b}\,p^2\,\theta^2}{\Psi^2} - \frac{6\,u^b\,u_a\,p'\,p'_{;b}\,\theta^2}{\Psi^2} + \frac{6\,u^b\,u_a\,p^2\,\Psi_{;b}\,\theta^2}{\Psi^3} - \frac{6\,u^b\,u_a\,p^2\,\theta\,\theta_{;b}}{\Psi^2} \tag{1.9}$$

and now some identities:

> temp9 := expand(TEDS(`p'`[-B]·u[b]=-`p''`θ(μ+p), temp8)) : T(%);

$$u^b\,\theta_{;a;b} = \frac{6\,\theta^3\,p''\,\mu\,p'\,u_a}{\Psi^2} + \frac{6\,\theta^3\,p''\,p\,p'\,u_a}{\Psi^2} - \frac{3\,\theta^2\,p''\,\mu\,\omega_a}{\Psi} - \frac{3\,\theta^2\,p''\,p\,\omega_a}{\Psi} - \frac{3\,u^b\,u_{a;b}\,p^2\,\theta^2}{\Psi^2} - \frac{6\,u^b\,u_a\,p^2\,\theta\,\theta_{;b}}{\Psi^2} + \frac{6\,u^b\,u_a\,p^2\,\Psi_{;b}\,\theta^2}{\Psi^3} + \frac{3\,u^b\,p'\,\theta\,\omega_{a;b}}{\Psi} + \frac{3\,u^b\,p'\,\theta_{;b}\,\omega_a}{\Psi} - \frac{3\,u^b\,p'\,\theta\,\omega_a\,\Psi_{;b}}{\Psi^2} \tag{1.10}$$

> temp10 := expand(TEDS(du[-b]·omega[b]=Psi·ω², expand(TEDS(omega[-a,-B]·u[b]

$= \theta \cdot \text{omega}[-a] \cdot \grave{}p'\grave{} - \frac{2}{3} \cdot \theta \cdot \omega[-a] + u[-a] \cdot du[-b] \cdot \omega[b], expand(TEDS(\text{Psi}[-B] \cdot u[b] = rhs(eq[65]), expand(TEDS(\text{theta}[-B] \cdot u[b] = rhs(eq[69]), expand(TEDS(u[b] \cdot u[-a, -B] = \text{Psi} \cdot \text{omega}[-a], temp9)))))))))) : T(\%);$

$$u^{b} \theta_{;a;b} = 3 p' \theta u_{a} \omega^{2} + \frac{2 \theta^{3} p'^{2} u_{a}}{\Psi^{2}} - \frac{3 \theta^{2} p' \omega_{a}}{\Psi} \quad \textbf{(1.11)}$$

where we have used:

$> eq[65] : T(\%);$

$$dotPsi = \left(- \frac{p'' \mu}{p'} - \frac{p'' p}{p'} + \frac{3 p'^{2}}{\Psi^{2}} + \frac{1}{3} \right) \Psi \theta \quad \textbf{(1.12)}$$

$> \text{omega}[-a, -B] \cdot u[b] = \theta \cdot \text{omega}[-a] \cdot \grave{}p'\grave{} - \frac{2}{3} \cdot \theta \cdot \omega[-a] + u[-a] \cdot du[-b] \cdot \omega[b] : T(\%);$

$$\omega_{a;b} u^{b} = p' \theta \omega_{a} - \frac{2}{3} \theta \omega_{a} + u_{a} du_{b} \omega^{b} \quad \textbf{(1.13)}$$

>

Looking at temp7 with the assumptions:

$> temp11 := expand(TEDS(temp3, TEDS(du[f] = \text{Psi} \cdot \text{omega}[f], TEDS(\text{omega}[f, -a] \cdot \text{theta}[-F] = 0, temp7)))) : T(\%);$

$$dottheta_{;a} = \theta_{;f} u_{a} \Psi \omega^{f} - \frac{2}{3} \theta u_{a} \omega^{2} + \frac{1}{9} \theta^{3} u_{a} + \frac{3 \theta^{3} p'^{2} u_{a}}{\Psi^{2}} + \frac{1}{6} \theta u_{a} \mu + \frac{1}{2} \theta u_{a} p - \frac{1}{3} \theta u_{a} du^{e}{}_{;e} - \frac{3 \theta^{2} p' \omega_{a}}{\Psi} + 4 \omega \omega_{;a} + du^{e}{}_{;e;a} - \frac{1}{2} \mu_{;a} - \frac{3}{2} p_{;a} \quad \textbf{(1.14)}$$

where we have used:

$> temp12 := subs(a = f, A = F, TEDS(\text{omega}[a] \cdot u[-a] = 0, expand(\text{omega}[a] \cdot eq[66]))) : T(\%);$

$$\omega^{f} \theta_{;f} = \frac{3 \omega^{f} p' \theta \omega_{f}}{\Psi} \quad \textbf{(1.15)}$$

$> temp13 := subs\left(a = -a, A = -A, isolate\left(TEDS\left(du[a] = \text{Psi} \cdot \text{omega}[a], TEDS\left(p[-B] \cdot u[b] = -\grave{}p'\grave{} \cdot \theta \cdot (\mu + p), subs(Absorbg(TEDS(P[a, b] = g[a, b] + u[a] \cdot u[b], eq[31])))\right)\right), p[A]\right)\right) : T(\%);$

0, "not a tensor"

$$p_{;a} = \mu p' \theta u_{a} + p p' \theta u_{a} - \Psi \mu \omega_{a} - \Psi \omega p_{a} \quad \textbf{(1.16)}$$

$> convert(temp13, string);$

"p[-A] = mu*`p'`*theta*u[-a]+p*`p'`*theta*u[-a]-Psi*mu*omega[-a]-Psi*p*omega[-a]" **(1.17)**

and also

$> temp14 := \text{mu}[-a] = \frac{rhs(temp13)}{\grave{}p'\grave{}} : T(\%);$

$$\mu_a = \frac{\mu\, p'\,\theta\, u_a + p\, p'\,\theta\, u_a - \Psi\,\mu\,\omega_a - \Psi\,\omega\, p_a}{p'} \qquad \mathbf{(1.18)}$$

>

> dotp =-p'θ (μ + p)

$$dotp = -\left(\frac{d}{dx}\, p(x)\right)\theta\,(\mu + p(x)) \qquad \mathbf{(1.19)}$$

> temp15 := expand(TEDS(temp14, expand(TEDS(temp13, expand(TEDS(temp12, temp11)))))) : T(%);

$$dottheta_{;a} = 3\, p'\,\theta\,\omega_f u_a\,\omega^f - \frac{2}{3}\,\theta\, u_a\,\omega^2 + \frac{1}{9}\,\theta^3 u_a + \frac{3\,\theta^3 p^2 u_a}{\Psi^2} + \frac{1}{6}\,\theta\, u_a\,\mu + \frac{1}{2}\,\theta\, u_a\, p - \frac{1}{3}\,\theta\, u_a\, du^e{}_{;e} - \frac{3\,\theta^2 p'\,\omega_a}{\Psi} + 4\,\omega\,\omega_{;a} + du^e{}_{;e\,;a} - \frac{1}{2}\,\mu_{;a} - \frac{3}{2}\,\mu\, p'\,\theta\, u_a - \frac{3}{2}\, p\, p'\,\theta\, u_a + \frac{3}{2}\,\Psi\,\mu\,\omega_a + \frac{3}{2}\,\Psi\, p\,\omega_a \qquad \mathbf{(1.20)}$$

so we have

> temp16 := rhs(temp15) − rhs(temp10) = 0 : T(%);

$$3\, p'\,\theta\,\omega_f u_a\,\omega^f - \frac{2}{3}\,\theta\, u_a\,\omega^2 + \frac{1}{9}\,\theta^3 u_a + \frac{\theta^3 p^2 u_a}{\Psi^2} + \frac{1}{6}\,\theta\, u_a\,\mu + \frac{1}{2}\,\theta\, u_a\, p - \frac{1}{3}\,\theta\, u_a\, du^e{}_{;e} + 4\,\omega\,\omega_{;a} + du^e{}_{;e\,;a} - \frac{1}{2}\,\mu_{;a} - \frac{3}{2}\,\mu\, p'\,\theta\, u_a - \frac{3}{2}\, p\, p'\,\theta\, u_a + \frac{3}{2}\,\Psi\,\mu\,\omega_a + \frac{3}{2}\,\Psi\, p\,\omega_a - 3\, p'\,\theta\, u_a\,\omega^2 = 0 \qquad \mathbf{(1.21)}$$

and contracting with omega[a]:

> temp17 := expand(TEDS(omega[a]·omega[−a] = ω², expand(TEDS(omega[a]·u[−a] = 0, expand(omega[a]·temp16))))) : T(%);

$$4\,\omega^a\,\omega\,\omega_{;a} + \omega^a\, du^e{}_{;e\,;a} - \frac{1}{2}\,\omega^a\,\mu_{;a} + \frac{3}{2}\,\Psi\,\mu\,\omega^2 + \frac{3}{2}\,\Psi\, p\,\omega^2 = 0 \qquad \mathbf{(1.22)}$$

> cod(omega·omega[a], −a) : T(%);

$$\omega\,\omega^a{}_{;a} + \omega_{;a}\, omega[-A]^a \qquad \mathbf{(1.23)}$$

> temp18 := du[e, −E] = cod(Psi·omega[e], −e) : T(%);

$$du^e{}_{;e} = \Psi\,\omega^e{}_{;e} + \Psi_{;e}\,\omega^e \qquad \mathbf{(1.24)}$$

> temp19 := TEDS(omega[e, −E] = 2·Psi·ω², temp18) : T(%);

$$du^e{}_{;e} = 2\,\Psi^2\,\omega^2 + \Psi_{;e}\,\omega^e \qquad \mathbf{(1.25)}$$

> convert(temp19, string);

(1.26)

"du[e,-E] = 2*Psi^2*omega^2+Psi[-E]*omega[e]" **(1.26)**

convert(temp19,string);

> #temp20:=cod(Psi[-E],-a) : T(%);

>

this does not ppear to get anywhere.....so we move to contratcting temp6 with u[b]

> temp21 := expand(u[b]·temp6) : T(%);

$$u^{b}\theta_{;b\,;a} = \frac{3\,u^{b}p'_{;a}\theta\,\omega_{b}}{\Psi} + \frac{3\,u^{b}p'\theta_{;a}\omega_{b}}{\Psi} + \frac{3\,u^{b}p'\theta\,\omega_{b\,;a}}{\Psi} - \frac{3\,u^{b}p'\theta\,\omega_{b}\Psi_{;a}}{\Psi^2} - \frac{3\,u^{b}u_{b\,;a}p^2\theta^2}{\Psi^2} - \frac{6\,u^{b}u_{b}p'p'_{;a}\theta^2}{\Psi^2} + \frac{6\,u^{b}u_{b}p^2\Psi_{;a}\theta^2}{\Psi^3} - \frac{6\,u^{b}u_{b}p^2\theta\,\theta_{;a}}{\Psi^2} \quad (1.27)$$

with some identities:

> temp22 := expand(TEDS(u[b]·u[-b]=-1, expand(TEDS(u[b]·u[-b,-A]=0, expand(TEDS(u[b]·omega[-b]=0, temp21)))))) : T(%);

$$u^{b}\theta_{;b\,;a} = \frac{3\,u^{b}p'\theta\,\omega_{b\,;a}}{\Psi} + \frac{6\,p^2\theta\,\theta_{;a}}{\Psi^2} + \frac{6\,p'p'_{;a}\theta^2}{\Psi^2} - \frac{6\,p^2\Psi_{;a}\theta^2}{\Psi^3} \quad (1.28)$$

Now from

> u[b]·omega[-b]=0 : T(%);

$$u^{b}\omega_{b} = 0 \quad (1.29)$$

we have from SSSeq6, and assumptions

> temp23 := isolate(cod(u[b]·omega[-b]=0,-a), u[b]·omega[-b,-A]) : T(%);

$$u^{b}\omega_{b\,;a} = -u^{b}{}_{;a}\,\omega_{b} \quad (1.30)$$

> temp24 := subs(a=c, A=C, b=a, B=A, c=-b, C=-B, subs(sigma=0, eq[6])) : T(%);

$$u^{b}{}_{;a} = \frac{1}{3}\,\theta\,P^{b}{}_{a} + \omega^{b}{}_{a} - du^{b}\,u_{a} \quad (1.31)$$

> temp25 := TEDS(P[b,-a]·omega[-b]=omega[-a], TEDS(du[b]·omega[-b]=Psi·ω², expand(TEDS(omega[-b]·omega[b,-a]=0, expand(TEDS(temp24, temp23)))))) : T(%);

$$u^{b}\omega_{b\,;a} = -\frac{1}{3}\,\theta\,\omega_{a} + u_{a}\Psi\,\omega^2 \quad (1.32)$$

> temp26 := expand(TEDS(temp25, temp22)) : T(%);

$$u^{b}\theta_{;b\,;a} = 3\,p'\theta\,u_{a}\omega^2 - \frac{\theta^2 p'\omega_{a}}{\Psi} + \frac{6\,p^2\theta\,\theta_{;a}}{\Psi^2} + \frac{6\,p'p'_{;a}\theta^2}{\Psi^2} - \frac{6\,p^2\Psi_{;a}\theta^2}{\Psi^3} \quad (1.33)$$

from kinematic quantities:

$$p'_{;b}=p''\,PU\,u_b\,\theta-\frac{p''\,PU\,du_b}{p'}$$

```
> temp27 := `p'`[-A] = `p''`·PU·u[-a]·theta − `p''`·PU·du[-a]/`p'` : T(%);
```

$$p'_{;a}=p''\,PU\,u_a\,\theta-\frac{p''\,PU\,du_a}{p'} \tag{1.34}$$

```
> temp28 := TEDS(temp27, temp26) : T(%);
```

$$u^{b}\,\theta_{;b\,;a}=\frac{1}{\Psi^3}\left(\theta\left(6\,PU\,\Psi\,p'\,p''\,\theta^2\,u_a+3\,\Psi^3\,\omega^2\,p'\,u_a-6\,PU\,\Psi\,du\,p''\,\theta_a-\Psi^2\,\omega\,p'\,\theta_a+6\,\Psi\,p^2\,\theta_{;a}-6\,\Psi\,p^2\,\theta_{;a}\right)\right) \tag{1.35}$$

and so we have:

```
> temp29 := expand(Ψ^3·expand(rhs(temp10) − rhs(temp28) = 0)) : T(%);
```

$$-6\,PU\,\Psi\,p'\,p''\,\theta^3\,u_a+2\,\Psi\,p^2\,\theta^3\,u_a+6\,PU\,\Psi\,du\,p''\,\theta^2{}_a-2\,\Psi^2\,\omega\,p'\,\theta^2{}_a-6\,\Psi\,p^2\,\theta\,\theta_{;a}+6\,\Psi\,p^2\,\theta^2{}_{;a}=0 \tag{1.36}$$

contracting with omega[a]:

```
> temp30 := 1/2 (expand(TEDS(omega[a]·omega[-a] = ω^2, expand(TEDS(omega[a]·u[-a]
      = 0, expand(omega[a]·temp29)))))) : T(%);
```

$$3\,PU\,\Psi\,p''\,\theta^2\,du_a\,\omega^{a}-\Psi^2\,p'\,\theta^2\,\omega^2-3\,\Psi\,p^2\,\theta\,\omega^{a}\,\theta_{;a}+3\,p^2\,\theta^2\,\Psi_{;a}\,\omega^{a}=0 \tag{1.37}$$

from SSSeq66:

```
> temp31 := TEDS(omega[a]·u[-a] = 0, expand(omega[a]·eq[66])) : T(%);
```

$$\omega^{a}\,\theta_{;a}=\frac{3\,\omega^{a}\,p'\,\theta\,\omega_a}{\Psi} \tag{1.38}$$

```
> temp32 := factor(TEDS(omega[a]·omega[-a] = ω^2, expand(TEDS(du[-a]·omega[a] = Psi
      ·ω^2, expand(TEDS(temp31, temp30)))))) : T(%);
```

$$\theta^2\left(3\,PU\,\Psi^2\,\omega^2\,p''-\Psi^2\,\omega^2\,p'-9\,\omega^2\,p^3+3\,p^2\,\Psi_{;a}\,\omega^{a}\right)=0 \tag{1.39}$$

so we have theta=0 OR

```
> temp33 := temp32/θ^2 : T(%);
```

$$3\,PU\,\Psi^2\,\omega^2\,p''-\Psi^2\,\omega^2\,p'-9\,\omega^2\,p^3+3\,p^2\,\Psi_{;a}\,\omega^{a}=0 \tag{1.40}$$

```
> temp34 := isolate(temp33, 3·`p'`^2·Psi[-A]·omega[a]) : T(%);
```

$$3\,p^2\,\Psi_{;a}\,\omega^{a}=-3\,PU\,\Psi^2\,\omega^2\,p''+\Psi^2\,\omega^2\,p'+9\,\omega^2\,p^3 \tag{1.41}$$

> $temp35 := \frac{collect\left(temp34, \left[\omega^2\right]\right)}{3 \cdot \Psi^2 \cdot `p'`} : T(\%);$

$$\frac{p' \Psi_{;a} \omega^{a}}{\Psi^2} = \frac{1}{3} \frac{\left(-3\, PU \Psi^2 p'' + \Psi^2 p' + 9\, p'^3\right) \omega^2}{\Psi^2 p'} \quad \textbf{(1.42)}$$

which is SSSeq70

> $convert(temp35, string);$

"1/Psi^2*`p'`*Psi[-A]*omega[a] = 1/3/Psi^2/`p'`*(-3*PU*Psi^2*`p''`+Psi^2*`p'`+9*`p'`^3)*omega^2" **(1.43)**

>