

> restart;with(Riemann):with(TensorPack): with(Canon):CDF(0): CDS(index):

Chapter XX Tensor analysis using indices - Senovilla et al. - Shearfree for acceleration parallel to vorticity if $\sigma_{ab}=0 \Rightarrow \omega\Theta=0$

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file 2d:eq62

> read "EFE": read "SFE": read "fids": read "Seneqs80":

proof of eq62:

Combining eq55 and eq61 (with the orthogonality assumption):

$$\begin{aligned} > \text{eq55SSS} := P[a, -b] \cdot \text{dotdu}[b] = & \left(p' - \frac{1}{3} - (\mu + p) \cdot \left(\frac{p''}{p'} \right) \right) \cdot \theta \cdot \text{du}[a] \\ & + \omega[a, -b] \cdot \text{du}[b] + p' \cdot P[a, b] \cdot \theta[-B]: T(\%); \\ P^a_b \text{dotdu}^b = & \left(p' - \frac{1}{3} - \frac{(\mu + p) p''}{p'} \right) \theta \text{du}^a + \text{du}^b \omega^a_b + p' P^a_b \theta_{;b} \end{aligned} \quad (1.1)$$

> temp := collect(TEDS($\text{du}[b] \cdot \omega[a, -b] = 0$, eq55SSS), [theta, du[a]]): T(%);

$$P^a_b \text{dotdu}^b = -\frac{1}{3} \frac{(3\mu p'' + 3p p'' - 3p^2 + p') \text{du}^a \theta}{p'} + p' P^a_b \theta_{;b} \quad (1.2)$$

> temp1 := subs(a=-a, eq[61]): T(%);

$$P^a_b \theta_{;b} = \frac{3p' \theta \omega^a}{\Psi} + \frac{3}{2} \frac{\Psi^{;d} \omega^a_d}{\Psi} \quad (1.3)$$

> temp2 := TEDS($\omega[a] = \frac{\text{du}[a]}{\Psi}, \text{temp1}$): T(%);

$$P^a_b \theta_{;b} = \frac{3}{2} \frac{\Psi^{;d} \omega^a_d \Psi + 2p' \theta \text{du}^a}{\Psi^2} \quad (1.4)$$

> temp3 := collect(TEDS(temp2, temp), [theta, du[a]]): T(%);

$$\begin{aligned} P^a_b \text{dotdu}^b = & -\frac{1}{6} \frac{(6\Psi^2 \mu p'' + 6\Psi^2 p p'' - 6\Psi^2 p^2 + 2\Psi^2 p' - 18p^3) \text{du}^a \theta}{\Psi^2 p'} \\ & + \frac{3}{2} \frac{p' \Psi^{;d} \omega^a_d}{\Psi} \end{aligned} \quad (1.5)$$

which is eq62

> convert(temp3, string);

$$\begin{aligned} "P[a,-b]*dotdu[b] = & -1/6*(6*\Psi^2*\mu*p'' + 6*\Psi^2*p*p'' - 6*\Psi^2*p^2 + 2*\Psi^2*p' - 18*p^3)/\Psi^2 \\ & *du[a]*theta + 3/2*p'*\Psi[D]/\Psi*\omega[a,-d]" \end{aligned} \quad (1.6)$$

> eq[62] := temp3: T(%);

$$\begin{aligned}
P^a{}_b \dot{d}u^b &= -\frac{1}{6} \frac{(6\Psi^2 \mu p'' + 6\Psi^2 p p'' - 6\Psi^2 p^2 + 2\Psi^2 p' - 18p^3) du^a \theta}{\Psi^2 p'} \quad (1.7) \\
&\quad + \frac{3}{2} \frac{p' \Psi^{,d} \omega^a{}_d}{\Psi}
\end{aligned}$$

> eq[62] := parse("P[a,-b]*dotdu[b]=1/6*(6*psi^2*p'^^2-6*psi^2*p''*mu-6*psi^2*p''*p+18*p'^^2*p*'`-2*p'*psi^2)/p'/psi^2*du[a]^theta+3/2*p'*Psi[D]*Psi*omega[a,-d]/psi^2":T(%);

$$\begin{aligned}
P^a{}_b \dot{d}u^b &= \frac{1}{6} \frac{(18' p p^2 - 6 \mu p'' \Psi^2 - 6 p p'' \Psi^2 + 6 p^2 \Psi^2 - 2 p' \Psi^2) du^a \theta}{p' \Psi^2} \quad (1.8) \\
&\quad + \frac{3}{2} \frac{p' \Psi^{,d} \Psi \omega^a{}_d}{\Psi^2}
\end{aligned}$$

=>