```
> restart;with(Riemann):with(TensorPack): with(Canon):CDF(0): CDS(index):
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for
acceleration parallel to vorticity

if $\sigma_{a\,b}$=0 =>$\omega\Theta$=0
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file 2d:eq59

```
> read "EFE" : read "SFE" :read "fids" :read "Seneqs80" :
```

proof of eq55:
We commence with time dilation of equation 56:

```
> eq[59] := P[a,-b]·dotomega[b] = (`p'` - 2/3)·theta·omega[a] : T(%);
```

$$P^{a}{}_{b}\,dotomega^{b} = \left(p' - \frac{2}{3}\right)\theta\,\omega^{a} \tag{1.1}$$

proof:
****

We commence with eq25:

```
> temp := eq[25] : T(%);
```

$$P^{a}{}_{b}\,dotomega^{b} + \frac{2}{3}\,\theta\,\omega^{a} - \frac{1}{2}\,\eta^{a\,b\,c\,d}\,u_{b}\,du_{c\,;d} = 0 \tag{1.2}$$

```
> eq[54] : T(%);
```

$$-\frac{1}{2}\,P_{a}{}^{c}\,P_{b}{}^{d}\,du_{d\,;c} + \frac{1}{2}\,P_{b}{}^{c}\,P_{a}{}^{d}\,du_{d\,;c} = p'\,\theta\,\omega_{a\,b} \tag{1.3}$$

```
> eq[11] : T(%);
```

$$\omega_{a\,b} = \eta_{a\,b\,e\,f}\,\omega^{e}\,u^{f} \tag{1.4}$$

and we define

```
> temp2 := TEDS(eq[11], temp) : T(%);
```

$$P^{a}{}_{b}\,dotomega^{b} + \frac{2}{3}\,\theta\,\omega^{a} - \frac{1}{2}\,\eta^{a\,b\,c\,d}\,u_{b}\,du_{c\,;d} = 0 \tag{1.5}$$

```
> temp3 := TEDS(eq[11], eq[54]) : T(%);
```

(1.6)

$$-\frac{1}{2} P_a{}^c P_b{}^d du_{d;c} + \frac{1}{2} P_b{}^c P_a{}^d du_{d;c} = p' \theta \eta_{abef} \omega^e u^f \quad (1.6)$$

> temp4 := Absorbg(TEDS(P[-b, c] = g[-b, c] + u[-b]·u[c], temp3)) : T(%);

$$-\frac{1}{2} P_a{}^c P_b{}^d du_{d;c} + \frac{1}{2} P_a{}^d du_{d;c} u^c u_b + \frac{1}{2} P_a{}^d du_{d;b} = p' \theta \eta_{abef} \omega^e u^f \quad (1.7)$$

> temp5 := Absorbg(TEDS(P[-a, d] = g[-a, d] + u[-a]·u[d], temp4)) : T(%);

$$-\frac{1}{2} P_a{}^c P_b{}^d du_{d;c} + \frac{1}{2} du_{d;c} u^c u^d u_a u_b + \frac{1}{2} du_{a;c} u^c u_b + \frac{1}{2} du_{d;b} u^d u_a + \frac{1}{2} du_{a;b} = p' \theta \eta_{abef} \omega^e u^f \quad (1.8)$$

> temp6 := Absorbg(TEDS(P[-a, c] = g[-a, c] + u[-a]·u[c], temp5)) : T(%);

$$-\frac{1}{2} P_b{}^d du_{d;c} u^c u_a - \frac{1}{2} P_b{}^d du_{d;a} + \frac{1}{2} du_{d;c} u^c u^d u_a u_b + \frac{1}{2} du_{a;c} u^c u_b + \frac{1}{2} du_{d;b} u^d u_a + \frac{1}{2} du_{a;b} = p' \theta \eta_{abef} \omega^e u^f \quad (1.9)$$

> temp7 := Absorbg(TEDS(P[-b, d] = g[-b, d] + u[-b]·u[d], temp6)) : T(%);

$$-\frac{1}{2} du_{b;c} u^c u_a - \frac{1}{2} du_{d;a} u^d u_b - \frac{1}{2} du_{b;a} + \frac{1}{2} du_{a;c} u^c u_b + \frac{1}{2} du_{d;b} u^d u_a + \frac{1}{2} du_{a;b} = p' \theta \eta_{abef} \omega^e u^f \quad (1.10)$$

multipying by eta

> temp8 := eta[a, b, g, h]·temp7 : T(%);

$$\eta^{abgh} \left(-\frac{1}{2} du_{b;c} u^c u_a - \frac{1}{2} du_{d;a} u^d u_b - \frac{1}{2} du_{b;a} + \frac{1}{2} du_{a;c} u^c u_b + \frac{1}{2} du_{d;b} u^d u_a + \frac{1}{2} du_{a;b} \right) = \eta^{abgh} p' \theta \eta_{abef} \omega^e u^f \quad (1.11)$$

> temp9 := Absorbd(Absorbd(expand(TEDS(eta[a, b, g, h]·eta[-a, -b, -e, -f] = -4 ·antisymm(delta[g, -e]·delta[h, -f], g, h), temp8)))) : T(%);

$$-\frac{1}{2} \eta^{abgh} du_{b;c} u^c u_a - \frac{1}{2} \eta^{abgh} du_{d;a} u^d u_b - \frac{1}{2} \eta^{abgh} du_{b;a} + \frac{1}{2} \eta^{abgh} du_{a;c} u^c u_b + \frac{1}{2} \eta^{abgh} du_{d;b} u^d u_a + \frac{1}{2} \eta^{abgh} du_{a;b} = -2 p' \theta \omega^g u^h + 2 p' \theta \omega^h u^g \quad (1.12)$$

> temp10 := expand(u[-g]·temp9) : T(%);

$$-\frac{1}{2} u_g \eta^{abgh} du_{b;c} u^c u_a - \frac{1}{2} u_g \eta^{abgh} du_{d;a} u^d u_b - \frac{1}{2} u_g \eta^{abgh} du_{b;a} + \frac{1}{2} u_g \eta^{abgh} du_{a;c} u^c u_b + \frac{1}{2} u_g \eta^{abgh} du_{d;b} u^d u_a \quad (1.13)$$

$$+\frac{1}{2}u_g\eta^{abgh}du_{a;b}=-2p'\theta\omega^g u^h u_g+2p'\theta\omega^h u^g u_g$$

```
> temp11 := expand(TEDS(u[-g]·omega[g]=0, temp10)) : T(%);
```

$$-\frac{1}{2}u_g\eta^{abgh}du_{b;c}u^c u_a-\frac{1}{2}u_g\eta^{abgh}du_{d;a}u^d u_b-\frac{1}{2}u_g\eta^{abgh}du_{b;a}+\frac{1}{2}u_g\eta^{abgh}du_{a;c}u^c u_b+\frac{1}{2}u_g\eta^{abgh}du_{d;b}u^d u_a+\frac{1}{2}u_g\eta^{abgh}du_{a;b}=2p'\theta\omega^h u^g u_g \quad \textbf{(1.14)}$$

```
> temp12 := expand(TEDS(u[-g]·u[g]=-1, temp11)) : T(%);
```

$$-\frac{1}{2}u_g\eta^{abgh}du_{b;c}u^c u_a-\frac{1}{2}u_g\eta^{abgh}du_{d;a}u^d u_b-\frac{1}{2}u_g\eta^{abgh}du_{b;a}+\frac{1}{2}u_g\eta^{abgh}du_{a;c}u^c u_b+\frac{1}{2}u_g\eta^{abgh}du_{d;b}u^d u_a+\frac{1}{2}u_g\eta^{abgh}du_{a;b}=-2p'\theta\omega^h \quad \textbf{(1.15)}$$

```
> temp13 := expand(TEDS(eta[a, b, g, h]·u[-g]·u[-b]=0, temp12)) : T(%);
```

$$-\frac{1}{2}u_g\eta^{abgh}du_{b;c}u^c u_a-\frac{1}{2}u_g\eta^{abgh}du_{b;a}+\frac{1}{2}u_g\eta^{abgh}du_{d;b}u^d u_a+\frac{1}{2}u_g\eta^{abgh}du_{a;b}=-2p'\theta\omega^h \quad \textbf{(1.16)}$$

```
> temp14 := expand(TEDS(eta[a, b, g, h]·u[-g]·u[-a]=0, temp13)) : T(%);
```

$$-\frac{1}{2}u_g\eta^{abgh}du_{b;a}+\frac{1}{2}u_g\eta^{abgh}du_{a;b}=-2p'\theta\omega^h \quad \textbf{(1.17)}$$

```
> temp15 := expand(TEDS(eta[a, b, g, h]·du[-b,-A]=-eta[a, b, g, h]·du[-a,-B], temp14)) :
    T(%);
```

$$u_g\eta^{abgh}du_{a;b}=-2p'\theta\omega^h \quad \textbf{(1.18)}$$

```
> temp16 := subs(g=d, b=c, B=C, a=b, h=a, expand(TEDS(eta[a, b, g, h]=-eta[h, a, b,
    g], temp15))) : T(%);
```

$$-u_d du_{b;c}\eta^{abcd}=-2p'\theta\omega^a \quad \textbf{(1.19)}$$

re − arranging the indices:

```
> temp17 := -expand(TEDS(u[-d]·du[-b,-C]=u[-b]·du[-c,-D], temp16)) : T(%);
```

$$\eta^{abcd}u_b du_{c;d}=2p'\theta\omega^a \quad \textbf{(1.20)}$$

```
>
```

```
> temp18 := TEDS(temp17, eq[25]) : T(%);
```

$$P^a{}_b dotomega^b+\frac{2}{3}\theta\omega^a-p'\theta\omega^a=0 \quad \textbf{(1.21)}$$

```
> temp19 := factor(temp18 - op(2, op(1, temp18)) - op(3, op(1, temp18))) : T(%);
```

$$P^{a}{}_{b}\,dotomega^{b} = \frac{1}{3}\,(3\,p' - 2)\,\theta\,\omega^{a} \quad \textbf{(1.22)}$$

which is eq59a

> *eq*[59] := *temp19* : *T*(%);

$$P^{a}{}_{b}\,dotomega^{b} = \frac{1}{3}\,(3\,p' - 2)\,\theta\,\omega^{a} \quad \textbf{(1.23)}$$

> *convert*(*temp19*, *string*);

"P[a,-b]*dotomega[b] = 1/3*(3*`p'`-2)*theta*omega[a]" **(1.24)**

****
****

>

> *temp20* := *Absorbg*(*expand*(*TEDS*(*P*[*a*, -*b*] = *g*[*a*, -*b*] + *u*[*a*]·*u*[-*b*], *temp19*))) : *T*(%);

$$u^{a}\,u_{b}\,dotomega^{b} + dotomega^{a} = -\frac{2}{3}\,\theta\,\omega^{a} + p'\,\theta\,\omega^{a} \quad \textbf{(1.25)}$$

contracting with ω_{a}

> *temp21* := *expand*(omega[-*a*]·*temp20*) : *T*(%);

$$\omega_{a}\,u^{a}\,u_{b}\,dotomega^{b} + \omega_{a}\,dotomega^{a} = -\frac{2}{3}\,\omega_{a}\,\theta\,\omega^{a} + \omega_{a}\,p'\,\theta\,\omega^{a} \quad \textbf{(1.26)}$$

> *temp22* := *expand*(*TEDS*(omega[-*a*]·*u*[*a*] = 0, *temp21*)) : *T*(%);

$$\omega_{a}\,dotomega^{a} = -\frac{2}{3}\,\omega_{a}\,\theta\,\omega^{a} + \omega_{a}\,p'\,\theta\,\omega^{a} \quad \textbf{(1.27)}$$

> *temp23* := *expand*(*TEDS*(omega[-*a*]·omega[*a*] = omega·omega, *temp22*)) : *T*(%);

$$\omega_{a}\,dotomega^{a} = -\frac{2}{3}\,\theta\,\omega^{2} + p'\,\theta\,\omega^{2} \quad \textbf{(1.28)}$$

Now

> *temp24* := *dotT*(omega[-*a*]·omega[*a*] = omega·omega) : *T*(%);

$$\omega^{a}\,dotomega_{a} + \omega_{a}\,dotomega^{a} = 2\,\omega\,dotomega \quad \textbf{(1.29)}$$

> *temp25* := *expand*(*TEDS*(*dotomega*[-*a*]·omega[*a*] = *dotomega*[*a*]·omega[-*a*], *temp24*)) :
T(%);

$$2\,\omega_{a}\,dotomega^{a} = 2\,\omega\,dotomega \quad \textbf{(1.30)}$$

> *temp26* := $\dfrac{factor(expand(TEDS(temp25, temp23)))}{\text{omega}}$: *T*(%);

$$dotomega = \frac{1}{3}\,\omega\,\theta\,(3\,p' - 2) \quad \textbf{(1.31)}$$

(which assumes $\omega \neq 0$)
This is eq59b

>