```
> restart;with(Riemann):with(TensorPack): with(Canon):CDF(0): CDS(index):
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for
acceleration parallel to vorticity

if $\sigma_{a\,b}$=0 =>ωΘ=0
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file 2d:eqs 52-54 (including 56)

In this file we continue to follow the equations outlined by Senovilla et al. (2007) with the assumptions for acceleration parallel to vorticity for proofs of eqs 52-54, but also including eqs56-57 as part of the proof process.

```
> read "EFE" : read "SFE" :read "fids" :read "Seneqs31" :
>
```

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Equation 52 - this is the assumption of this part of the theorem

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> eq[52] := du = psi·omega : T(%);
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$$du = \psi\,\omega \tag{1.1}$$

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Equation 53 - defn of p': eqn of state

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> eq[53] := p`'` = dp/(d mu) : T(%);
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$$p\,' = \frac{dp}{d\,\mu} \tag{1.2}$$

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Equation 54

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> $eq[54] := antisymm(P[-b, c]\cdot P[-a, d]\cdot du[-d,-C],-b,-a) = `p'`\cdot \text{theta}\cdot \text{omega}[-a,-b] :$
> $T(\%);$

$$-\frac{1}{2} P_{a}{}^{c} P_{b}{}^{d} du_{d;c} + \frac{1}{2} P_{b}{}^{c} P_{a}{}^{d} du_{d;c} = p' \theta \omega_{ab} \quad \textbf{(1.3)}$$

proof:

> $eq[31] := (\text{mu} + p)\cdot du[a] + P[a, b]\cdot p[-B] = 0 : T(\%);$

$$(\mu + p)\, du^{a} + P^{a\,b} p_{;b} = 0 \quad \textbf{(1.4)}$$

> $eq[31] : T(\%);$

$$(\mu + p)\, du^{a} + P^{a\,b} p_{;b} = 0 \quad \textbf{(1.5)}$$

> $eq[30] : T(\%);$

$$dotmu + \theta\, (\mu + p) = 0 \quad \textbf{(1.6)}$$

from eq31:

> $temp := eq[31] - op(2, op(1, eq[31])) : T(\%);$

$$(\mu + p)\, du^{a} = -P^{a\,b} p_{;b} \quad \textbf{(1.7)}$$

> $temp2 := \frac{temp}{(\text{mu} + p)} : T(\%);$

$$du^{a} = -\frac{P^{a\,b} p_{;b}}{\mu + p} \quad \textbf{(1.8)}$$

> $temp3 := Absorbg(expand(TEDS(P[a, b] = g[a, b] + u[a]\cdot u[b], temp2))) : T(\%);$

$$du^{a} = -\frac{p_{;b} u^{a} u^{b}}{\mu + p} - \frac{p^{;a}}{\mu + p} \quad \textbf{(1.9)}$$

> $temp4 := TEDS(p[-B]\cdot u[b] = pdot, temp3) : T(\%);$

$$du^{a} = -\frac{u^{a} pdot + p^{;a}}{\mu + p} \quad \textbf{(1.10)}$$

Now from eq30

> $eq[30] : T(\%);$

$$dotmu + \theta\, (\mu + p) = 0 \quad \textbf{(1.11)}$$

> $temp5 := eq[30] - op(2, op(1, eq[30])) : T(\%);$

$$dotmu = -\theta\, (\mu + p) \quad \textbf{(1.12)}$$

> $eq[53] : T(\%);$

$$p' = \frac{dp}{d\mu} \quad \textbf{(1.13)}$$

Now from the chain rule:

> $temp6 := pdot = `p'`\cdot dotmu : T(\%);$

$$pdot = p'\, dotmu \quad \textbf{(1.14)}$$

>

i.e. dp/dt = dp/dμ · $d\mu/dt$

> temp7 := TEDS(temp6, temp4) : T(%);

$$du^{a} = -\frac{u^{a}\,p'\,dotmu + p^{;a}}{\mu + p} \quad (1.15)$$

> temp8 := factor(TEDS(temp5, temp7)) : T(%);

$$du^{a} = \frac{u^{a}\,p'\,\theta\,\mu + u^{a}\,p'\,\theta\,p - p^{;a}}{\mu + p} \quad (1.16)$$

It can be seen that, by collect u, p theta:

> temp9 := du[a] = \`p'\`·theta·u[a] − \`p'\`·mu[A] / (p + mu) : T(%);

$$du^{a} = u^{a}\,p'\,\theta - \frac{p'\,\mu^{;a}}{\mu + p} \quad (1.17)$$

which is equation 56

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It can easily be shown that (see p16 of notebook, to be written as proof eq57), using

> temp10 := antisymm(P[-b, c]·P[-a, d]·du[-d,-C],-b,-a) = \`p'\`·theta·antisymm(P[-b, c]·P[-a, d]·u[-d,-C],-b,-a) : T(%);

$$-\frac{1}{2}\,P_{a}{}^{c}P_{b}{}^{d}du_{d\,;c} + \frac{1}{2}\,P_{b}{}^{c}P_{a}{}^{d}du_{d\,;c} = p'\theta\left(-\frac{1}{2}\,P_{a}{}^{c}P_{b}{}^{d}u_{d\,;c} + \frac{1}{2}\,P_{b}{}^{c}P_{a}{}^{d}u_{d\,;c}\right) \quad (1.18)$$

or written as

> temp10b := P[-\`[b\`, c]·P[-\`a]\`, d]·du[-d,-C] = P[-\`[b\`, c]·P[-\`a]\`, d]·\`p'\`·theta·u[-d,-C] : T(%);

$$P_{[b}{}^{c}P_{a]}{}^{d}du_{d\,;c} = P_{[b}{}^{c}P_{a]}{}^{d}p'\,\theta\,u_{d\,;c} \quad (1.19)$$

which is eq57

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It is a simple subsitution of eq9 to lead to eq54:

> temp11 := subs(c = e, C = E, d = c, D = C, e = d, E = D, eq[9]) : T(%);

$$\omega_{a\,b} = -\frac{1}{2}\,P_{a}{}^{c}P_{b}{}^{d}u_{d\,;c} + \frac{1}{2}\,P_{b}{}^{c}P_{a}{}^{d}u_{d\,;c} \quad (1.20)$$

> temp11b := rhs(temp11) = lhs(temp11) : T(%);

$$-\frac{1}{2} P_a{}^c P_b{}^d u_{d;c} + \frac{1}{2} P_b{}^c P_a{}^d u_{d;c} = \omega_{ab} \quad \textbf{(1.21)}$$

> *temp12* := *TEDS*(*temp11b*, *temp10*) : *T*(%);

$$-\frac{1}{2} P_a{}^c P_b{}^d du_{d;c} + \frac{1}{2} P_b{}^c P_a{}^d du_{d;c} = p' \theta\, \omega_{ab} \quad \textbf{(1.22)}$$

which is eq54:

> *eq*[54] : *T*(%);

$$-\frac{1}{2} P_a{}^c P_b{}^d du_{d;c} + \frac{1}{2} P_b{}^c P_a{}^d du_{d;c} = p' \theta\, \omega_{ab} \quad \textbf{(1.23)}$$

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