```
> restart;
> with(Riemann):with(Canon):
> with(TensorPack) : CDF(0) : CDS(index) :
```

Chapter XX
Tensor analysis using indices - Senovilla et al. - Shearfree for dust
page 3

if $\sigma_{a\,b}$=0 =>$\omega\Theta$=0
Author: Peter Huf

HC52

```
> read "EFE" : read "SFE" :read "fids" :read "eqs2" :read "Seneqs2f" : read "dustvids" :
>
```

We aim to prove:

```
> HC[52] := parse("P[-a, b]*omega[c]*omega[-c,-B]=u[-a]*theta*`p'`*omega^2-2
      /3*omega^2*u[-a]*theta+omega*omega[-A]"):T(%);
```

$$P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b}=u_{a}\,\theta\,p'\,\omega^{2}-\frac{2}{3}\,\omega^{2}\,u_{a}\,\theta+\omega\,\omega_{;a} \tag{1.1}$$

```
>
```

****

We commence with the LHS:

```
> remainder := P[-a, b]·omega[c]·omega[-c,-B] : T(%);
```

$$P_{a}{}^{b}\,\omega^{c}\,\omega_{c\,;b} \tag{1.1.1}$$

```
> temp := omega[a] = 1/2·eta[a, b, c, d]·u[-b]·omega[-c,-d] : T(%);
```

$$\omega^{a}=\frac{1}{2}\,\eta^{a\,b\,c\,d}\,u_{b}\,\omega_{c\,d} \tag{1.1.2}$$

```
> temp2 := subs(b = i, c = j, d = k, a = c, temp) : T(%);
```

$$\omega^{c} = \frac{1}{2}\,\eta^{c\,i\,j\,k}\,u_{i}\,\omega_{j\,k} \tag{1.1.3}$$

```
> temp3 := subs(c=-c, i=-m, j=-n, k=-o, temp2) : T(%);
```

$$\omega_{c} = \frac{1}{2}\,\eta_{c\,m\,n\,o}\,u^{m}\,\omega^{n\,o} \tag{1.1.4}$$

```
> temp4 := cod(temp3, -b) : T(%);
```

$$\omega_{c\,;b} = \frac{1}{2}\,\eta_{c\,m\,n\,o\,;b}\,u^{m}\,\omega^{n\,o} + \frac{1}{2}\,\eta_{c\,m\,n\,o}\,u^{m}{}_{;b}\,\omega^{n\,o} + \frac{1}{2}\,\eta_{c\,m\,n\,o}\,u^{m}\,\omega^{n\,o}{}_{;b} \tag{1.1.5}$$

```
> temp5 := expand(TEDS(eta[-c,-m,-n,-o,-B]=0, temp4)) : T(%);
```

$$\omega_{c\,;b} = \frac{1}{2}\,\eta_{c\,m\,n\,o}\,u^{m}{}_{;b}\,\omega^{n\,o} + \frac{1}{2}\,\eta_{c\,m\,n\,o}\,u^{m}\,\omega^{n\,o}{}_{;b} \tag{1.1.6}$$

```
> temp6 := expand(TEDS(temp5, remainder)) : T(%);
```

$$\frac{1}{2}\,P_{a}{}^{b}\,\eta_{c\,m\,n\,o}\,\omega^{c}\,\omega^{n\,o}\,u^{m}{}_{;b} + \frac{1}{2}\,P_{a}{}^{b}\,\eta_{c\,m\,n\,o}\,\omega^{c}\,\omega^{n\,o}{}_{;b}\,u^{m} \tag{1.1.7}$$

```
> temp7 := expand(TEDS(temp2, temp6)) : T(%);
```

$$\frac{1}{4}\,P_{a}{}^{b}\,\eta_{c\,m\,n\,o}\,\omega^{n\,o}\,u^{m}{}_{;b}\,\eta^{c\,i\,j\,k}\,u_{i}\,\omega_{j\,k} + \frac{1}{4}\,P_{a}{}^{b}\,\eta_{c\,m\,n\,o}\,\omega^{n\,o}{}_{;b}\,u^{m}\,\eta^{c\,i\,j\,k}\,u_{i}\,\omega_{j\,k} \tag{1.1.8}$$

```
> temp8 := expand(TEDS(eta[c,i,j,k]·eta[-c,-m,-n,-o]=-6·antisymm(delta[i,-m]
    ·delta[j,-n]·delta[k,-o], i, k), temp7)) : T(%);
```

$$\begin{aligned}
&-\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{m}\,\delta^{j}{}_{n}\,\delta^{k}{}_{o}\,\omega^{n\,o}\,\omega_{j\,k}\,u_{i}\,u^{m}{}_{;b} \\
&+\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{m}\,\delta^{j}{}_{o}\,\delta^{k}{}_{n}\,\omega^{n\,o}\,\omega_{j\,k}\,u_{i}\,u^{m}{}_{;b} \\
&+\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{n}\,\delta^{j}{}_{m}\,\delta^{k}{}_{o}\,\omega^{n\,o}\,\omega_{j\,k}\,u_{i}\,u^{m}{}_{;b} \\
&-\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{n}\,\delta^{j}{}_{o}\,\delta^{k}{}_{m}\,\omega^{n\,o}\,\omega_{j\,k}\,u_{i}\,u^{m}{}_{;b} \\
&-\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{o}\,\delta^{j}{}_{m}\,\delta^{k}{}_{n}\,\omega^{n\,o}\,\omega_{j\,k}\,u_{i}\,u^{m}{}_{;b} \\
&+\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{o}\,\delta^{j}{}_{n}\,\delta^{k}{}_{m}\,\omega^{n\,o}\,\omega_{j\,k}\,u_{i}\,u^{m}{}_{;b} \\
&-\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{m}\,\delta^{j}{}_{n}\,\delta^{k}{}_{o}\,\omega_{j\,k}\,\omega^{n\,o}{}_{;b}\,u^{m}\,u_{i} \\
&+\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{m}\,\delta^{j}{}_{o}\,\delta^{k}{}_{n}\,\omega_{j\,k}\,\omega^{n\,o}{}_{;b}\,u^{m}\,u_{i} \\
&+\frac{1}{4}\,P_{a}{}^{b}\,\delta^{i}{}_{n}\,\delta^{j}{}_{m}\,\delta^{k}{}_{o}\,\omega_{j\,k}\,\omega^{n\,o}{}_{;b}\,u^{m}\,u_{i}
\end{aligned} \tag{1.1.9}$$

$$-\frac{1}{4} P_{a}{}^{b} \delta^{i}{}_{n} \delta^{j}{}_{o} \delta^{k}{}_{m} \omega_{j k} \omega^{n o}{}_{;b} u^{m} u_{i}$$

$$-\frac{1}{4} P_{a}{}^{b} \delta^{i}{}_{o} \delta^{j}{}_{m} \delta^{k}{}_{n} \omega_{j k} \omega^{n o}{}_{;b} u^{m} u_{i}$$

$$+\frac{1}{4} P_{a}{}^{b} \delta^{i}{}_{o} \delta^{j}{}_{n} \delta^{k}{}_{m} \omega_{j k} \omega^{n o}{}_{;b} u^{m} u_{i}$$

> *temp9* := *Absorbd*(*Absorbd*(*Absorbd*(*temp8*))) : *T*(%);

$$-\frac{1}{4} P_{a}{}^{b} \omega^{n o} \omega_{n o} u_{m} u^{m}{}_{;b} + \frac{1}{4} P_{a}{}^{b} \omega^{n o} \omega_{o n} u_{m} u^{m}{}_{;b} \qquad \textbf{(1.1.10)}$$

$$+\frac{1}{4} P_{a}{}^{b} \omega^{n o} \omega_{m o} u_{n} u^{m}{}_{;b} - \frac{1}{4} P_{a}{}^{b} \omega^{n o} \omega_{o m} u_{n} u^{m}{}_{;b}$$

$$-\frac{1}{4} P_{a}{}^{b} \omega^{n o} \omega_{m n} u_{o} u^{m}{}_{;b} + \frac{1}{4} P_{a}{}^{b} \omega^{n o} \omega_{n m} u_{o} u^{m}{}_{;b}$$

$$-\frac{1}{4} P_{a}{}^{b} \omega_{n o} \omega^{n o}{}_{;b} u^{m} u_{m} + \frac{1}{4} P_{a}{}^{b} \omega_{o n} \omega^{n o}{}_{;b} u^{m} u_{m}$$

$$+\frac{1}{4} P_{a}{}^{b} \omega_{m o} \omega^{n o}{}_{;b} u^{m} u_{n} - \frac{1}{4} P_{a}{}^{b} \omega_{o m} \omega^{n o}{}_{;b} u^{m} u_{n}$$

$$-\frac{1}{4} P_{a}{}^{b} \omega_{m n} \omega^{n o}{}_{;b} u^{m} u_{o} + \frac{1}{4} P_{a}{}^{b} \omega_{n m} \omega^{n o}{}_{;b} u^{m} u_{o}$$

start here

> *temp10* := *expand*(*TEDS*(*P*[−*a*, *b*] = *g*[−*a*, *b*] + *u*[−*a*]·*u*[*b*], *temp9*)) : *T*(%);

$$-\frac{1}{4} \omega^{n o} \omega_{n o} u^{b} u_{a} u_{m} u^{m}{}_{;b} - \frac{1}{4} g_{a}{}^{b} \omega^{n o} \omega_{n o} u_{m} u^{m}{}_{;b} \qquad \textbf{(1.1.11)}$$

$$+\frac{1}{4} \omega^{n o} \omega_{o n} u^{b} u_{a} u_{m} u^{m}{}_{;b} + \frac{1}{4} g_{a}{}^{b} \omega^{n o} \omega_{o n} u_{m} u^{m}{}_{;b}$$

$$+\frac{1}{4} \omega^{n o} \omega_{m o} u^{b} u_{a} u_{n} u^{m}{}_{;b} + \frac{1}{4} g_{a}{}^{b} \omega^{n o} \omega_{m o} u_{n} u^{m}{}_{;b}$$

$$-\frac{1}{4} \omega^{n o} \omega_{o m} u^{b} u_{a} u_{n} u^{m}{}_{;b} - \frac{1}{4} g_{a}{}^{b} \omega^{n o} \omega_{o m} u_{n} u^{m}{}_{;b}$$

$$-\frac{1}{4} \omega^{n o} \omega_{m n} u^{b} u_{a} u_{o} u^{m}{}_{;b} - \frac{1}{4} g_{a}{}^{b} \omega^{n o} \omega_{m n} u_{o} u^{m}{}_{;b}$$

$$+\frac{1}{4} \omega^{n o} \omega_{n m} u^{b} u_{a} u_{o} u^{m}{}_{;b} + \frac{1}{4} g_{a}{}^{b} \omega^{n o} \omega_{n m} u_{o} u^{m}{}_{;b}$$

$$-\frac{1}{4} \omega_{n o} \omega^{n o}{}_{;b} u^{b} u^{m} u_{a} u_{m} - \frac{1}{4} g_{a}{}^{b} \omega_{n o} \omega^{n o}{}_{;b} u^{m} u_{m}$$

$$+\frac{1}{4} \omega_{o n} \omega^{n o}{}_{;b} u^{b} u^{m} u_{a} u_{m} + \frac{1}{4} g_{a}{}^{b} \omega_{o n} \omega^{n o}{}_{;b} u^{m} u_{m}$$

$$+\frac{1}{4} \omega_{m o} \omega^{n o}{}_{;b} u^{b} u^{m} u_{a} u_{n} + \frac{1}{4} g_{a}{}^{b} \omega_{m o} \omega^{n o}{}_{;b} u^{m} u_{n}$$

$$-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}-\frac{1}{4}\,g_{a}{}^{b}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;b}\,u^{m}\,u_{n}$$

$$-\frac{1}{4}\,\omega_{m\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}-\frac{1}{4}\,g_{a}{}^{b}\,\omega_{m\,n}\,\omega^{n\,o}{}_{;b}\,u^{m}\,u_{o}$$

$$+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}+\frac{1}{4}\,g_{a}{}^{b}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;b}\,u^{m}\,u_{o}$$

> *temp11* := *Absorbg*(*temp10*) : *T*(%);

$$-\frac{1}{4}\,\omega^{n\,o}\,\omega_{n\,o}\,u^{b}\,u_{a}\,u_{m}\,u^{m}{}_{;b}-\frac{1}{4}\,\omega^{n\,o}\,\omega_{n\,o}\,u_{m}\,u^{m}{}_{;a}$$

$$+\frac{1}{4}\,\omega^{n\,o}\,\omega_{o\,n}\,u^{b}\,u_{a}\,u_{m}\,u^{m}{}_{;b}+\frac{1}{4}\,\omega^{n\,o}\,\omega_{o\,n}\,u_{m}\,u^{m}{}_{;a}$$

$$+\frac{1}{4}\,\omega^{n\,o}\,\omega_{m\,o}\,u^{b}\,u_{a}\,u_{n}\,u^{m}{}_{;b}+\frac{1}{4}\,\omega^{n\,o}\,\omega_{m\,o}\,u_{n}\,u^{m}{}_{;a}$$

$$-\frac{1}{4}\,\omega^{n\,o}\,\omega_{o\,m}\,u^{b}\,u_{a}\,u_{n}\,u^{m}{}_{;b}-\frac{1}{4}\,\omega^{n\,o}\,\omega_{o\,m}\,u_{n}\,u^{m}{}_{;a}$$

$$-\frac{1}{4}\,\omega^{n\,o}\,\omega_{m\,n}\,u^{b}\,u_{a}\,u_{o}\,u^{m}{}_{;b}-\frac{1}{4}\,\omega^{n\,o}\,\omega_{m\,n}\,u_{o}\,u^{m}{}_{;a}$$

$$+\frac{1}{4}\,\omega^{n\,o}\,\omega_{n\,m}\,u^{b}\,u_{a}\,u_{o}\,u^{m}{}_{;b}+\frac{1}{4}\,\omega^{n\,o}\,\omega_{n\,m}\,u_{o}\,u^{m}{}_{;a}$$

$$-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}$$

$$+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}$$

$$+\frac{1}{4}\,\omega_{m\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}+\frac{1}{4}\,\omega_{m\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$-\frac{1}{4}\,\omega_{m\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}-\frac{1}{4}\,\omega_{m\,n}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{o}$$

$$+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{o}$$

>

> *temp12* := *expand*(*TEDS*(omega[*n*, *o*]·*u*[−*n*] = 0, *temp11*)) : *T*(%);

$$-\frac{1}{4}\,\omega^{n\,o}\,\omega_{n\,o}\,u^{b}\,u_{a}\,u_{m}\,u^{m}{}_{;b}-\frac{1}{4}\,\omega^{n\,o}\,\omega_{n\,o}\,u_{m}\,u^{m}{}_{;a}$$

$$+\frac{1}{4}\,\omega^{n\,o}\,\omega_{o\,n}\,u^{b}\,u_{a}\,u_{m}\,u^{m}{}_{;b}+\frac{1}{4}\,\omega^{n\,o}\,\omega_{o\,n}\,u_{m}\,u^{m}{}_{;a}$$

$$-\frac{1}{4}\,\omega^{n\,o}\,\omega_{m\,n}\,u^{b}\,u_{a}\,u_{o}\,u^{m}{}_{;b}-\frac{1}{4}\,\omega^{n\,o}\,\omega_{m\,n}\,u_{o}\,u^{m}{}_{;a}$$

$$+\frac{1}{4}\,\omega^{n\,o}\omega_{n\,m}u^{b}u_{a}u_{o}u^{m}{}_{;b}+\frac{1}{4}\,\omega^{n\,o}\omega_{n\,m}u_{o}u^{m}{}_{;a}$$

$$-\frac{1}{4}\,\omega_{n\,o}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{m}-\frac{1}{4}\,\omega_{n\,o}\omega^{n\,o}{}_{;a}u^{m}u_{m}$$

$$+\frac{1}{4}\,\omega_{o\,n}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{m}+\frac{1}{4}\,\omega_{o\,n}\omega^{n\,o}{}_{;a}u^{m}u_{m}$$

$$+\frac{1}{4}\,\omega_{m\,o}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{n}+\frac{1}{4}\,\omega_{m\,o}\omega^{n\,o}{}_{;a}u^{m}u_{n}$$

$$-\frac{1}{4}\,\omega_{o\,m}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{n}-\frac{1}{4}\,\omega_{o\,m}\omega^{n\,o}{}_{;a}u^{m}u_{n}$$

$$-\frac{1}{4}\,\omega_{m\,n}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{o}-\frac{1}{4}\,\omega_{m\,n}\omega^{n\,o}{}_{;a}u^{m}u_{o}$$

$$+\frac{1}{4}\,\omega_{n\,m}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{o}+\frac{1}{4}\,\omega_{n\,m}\omega^{n\,o}{}_{;a}u^{m}u_{o}$$

> *temp13* := *expand*(*TEDS*(omega[*n*, *o*]·*u*[−*o*] = 0, *temp12*)) : *T*(%);

$$-\frac{1}{4}\,\omega^{n\,o}\omega_{n\,o}u^{b}u_{a}u_{m}u^{m}{}_{;b}-\frac{1}{4}\,\omega^{n\,o}\omega_{n\,o}u_{m}u^{m}{}_{;a} \qquad \textbf{(1.1.14)}$$

$$+\frac{1}{4}\,\omega^{n\,o}\omega_{o\,n}u^{b}u_{a}u_{m}u^{m}{}_{;b}+\frac{1}{4}\,\omega^{n\,o}\omega_{o\,n}u_{m}u^{m}{}_{;a}$$

$$-\frac{1}{4}\,\omega_{n\,o}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{m}-\frac{1}{4}\,\omega_{n\,o}\omega^{n\,o}{}_{;a}u^{m}u_{m}$$

$$+\frac{1}{4}\,\omega_{o\,n}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{m}+\frac{1}{4}\,\omega_{o\,n}\omega^{n\,o}{}_{;a}u^{m}u_{m}$$

$$+\frac{1}{4}\,\omega_{m\,o}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{n}+\frac{1}{4}\,\omega_{m\,o}\omega^{n\,o}{}_{;a}u^{m}u_{n}$$

$$-\frac{1}{4}\,\omega_{o\,m}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{n}-\frac{1}{4}\,\omega_{o\,m}\omega^{n\,o}{}_{;a}u^{m}u_{n}$$

$$-\frac{1}{4}\,\omega_{m\,n}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{o}-\frac{1}{4}\,\omega_{m\,n}\omega^{n\,o}{}_{;a}u^{m}u_{o}$$

$$+\frac{1}{4}\,\omega_{n\,m}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{o}+\frac{1}{4}\,\omega_{n\,m}\omega^{n\,o}{}_{;a}u^{m}u_{o}$$

> *temp14* := *expand*(*TEDS*(omega[−*m*, −*n*]·*u*[*m*] = 0, *temp13*)) : *T*(%);

$$-\frac{1}{4}\,\omega^{n\,o}\omega_{n\,o}u^{b}u_{a}u_{m}u^{m}{}_{;b}-\frac{1}{4}\,\omega^{n\,o}\omega_{n\,o}u_{m}u^{m}{}_{;a} \qquad \textbf{(1.1.15)}$$

$$+\frac{1}{4}\,\omega^{n\,o}\omega_{o\,n}u^{b}u_{a}u_{m}u^{m}{}_{;b}+\frac{1}{4}\,\omega^{n\,o}\omega_{o\,n}u_{m}u^{m}{}_{;a}$$

$$-\frac{1}{4}\,\omega_{n\,o}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{m}-\frac{1}{4}\,\omega_{n\,o}\omega^{n\,o}{}_{;a}u^{m}u_{m}$$

$$+\frac{1}{4}\,\omega_{o\,n}\omega^{n\,o}{}_{;b}u^{b}u^{m}u_{a}u_{m}+\frac{1}{4}\,\omega_{o\,n}\omega^{n\,o}{}_{;a}u^{m}u_{m}$$

$$+\frac{1}{4}\,\omega_{m\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}+\frac{1}{4}\,\omega_{m\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{o}$$

> *temp15* := *expand*(*TEDS*(*u*[−*m*]·*u*[*m*,−*A*] = 0, *temp14*)) : *T*(%);

$$-\frac{1}{4}\,\omega^{n\,o}\,\omega_{n\,o}\,u^{b}\,u_{a}\,u_{m}\,u^{m}{}_{;b}+\frac{1}{4}\,\omega^{n\,o}\,\omega_{o\,n}\,u^{b}\,u_{a}\,u_{m}\,u^{m}{}_{;b} \qquad \textbf{(1.1.16)}$$

$$-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}$$

$$+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}$$

$$+\frac{1}{4}\,\omega_{m\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}+\frac{1}{4}\,\omega_{m\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{o}$$

> *temp16* := *expand*(*TEDS*(*u*[−*m*]·*u*[*m*,−*B*] = 0, *temp15*)) : *T*(%);

$$-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m} \qquad \textbf{(1.1.17)}$$

$$+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}$$

$$+\frac{1}{4}\,\omega_{m\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}+\frac{1}{4}\,\omega_{m\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{o}$$

> *temp17* := *expand*(*TEDS*(*u*[*m*]·omega[−*m*,−*o*] = 0, *temp16*)) : *T*(%);

$$-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m} \qquad \textbf{(1.1.18)}$$

$$+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}$$

$$-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{n}-\frac{1}{4}\,\omega_{o\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{n}$$

$$+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{o}$$

```
> temp18 := expand(TEDS(u[m]·omega[-o,-m]=0, temp17)) : T(%);
```

$$-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{o}+\frac{1}{4}\,\omega_{n\,m}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{o} \tag{1.1.19}$$

```
> temp19 := expand(TEDS(u[m]·omega[-n,-m]=0, temp18)) : T(%);
```

$$-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}-\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m}+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u^{m}\,u_{a}\,u_{m}+\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a}\,u^{m}\,u_{m} \tag{1.1.20}$$

```
> temp20 := expand(TEDS(u[m]·u[-m]=-1, temp19)) : T(%);
```

$$\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u_{a}+\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a}-\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;b}\,u^{b}\,u_{a}-\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a} \tag{1.1.21}$$

```
> #temp21:=expand(TEDS(omega[n, o, -B]·u[b]=-2/3·theta·omega[n, o], temp20)) :
  T(%);
> temp21 := expand(TEDS(omega[n, o, -B]·u[b]=(`p'`-2/3)·theta·omega[n, o]+u[n]
  ·v[o]-u[o]·v[n], temp20)) : T(%);
```

$$\frac{1}{4}\,\omega_{n\,o}\,u_{a}\,\theta\,\omega^{n\,o}\,p'-\frac{1}{6}\,\omega_{n\,o}\,u_{a}\,\theta\,\omega^{n\,o}+\frac{1}{4}\,\omega_{n\,o}\,u_{a}\,u^{n}\,v^{o}-\frac{1}{4}\,\omega_{n\,o}\,u_{a}\,u^{o}\,v^{n}+\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a}-\frac{1}{4}\,\omega_{o\,n}\,u_{a}\,\theta\,\omega^{n\,o}\,p'+\frac{1}{6}\,\omega_{o\,n}\,u_{a}\,\theta\,\omega^{n\,o}-\frac{1}{4}\,\omega_{o\,n}\,u_{a}\,u^{n}\,v^{o}+\frac{1}{4}\,\omega_{o\,n}\,u_{a}\,u^{o}\,v^{n}-\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a} \tag{1.1.22}$$

```
>
> temp22 := expand(TEDS(omega[n, o]·omega[-n,-o]=2·omega·omega, temp21)) :
  T(%);
```

$$-\frac{1}{4}\,\omega_{o\,n}\,u_{a}\,\theta\,\omega^{n\,o}\,p'+\frac{1}{6}\,\omega_{o\,n}\,u_{a}\,\theta\,\omega^{n\,o}+\frac{1}{2}\,u_{a}\,\theta\,p'\,\omega^{2}-\frac{1}{3}\,\omega^{2}\,u_{a}\,\theta-\frac{1}{4}\,\omega_{o\,n}\,u_{a}\,u^{n}\,v^{o}+\frac{1}{4}\,\omega_{o\,n}\,u_{a}\,u^{o}\,v^{n}-\frac{1}{4}\,\omega_{o\,n}\,\omega^{n\,o}{}_{;a}+\frac{1}{4}\,\omega_{n\,o}\,u_{a}\,u^{n}\,v^{o}-\frac{1}{4}\,\omega_{n\,o}\,u_{a}\,u^{o}\,v^{n}+\frac{1}{4}\,\omega_{n\,o}\,\omega^{n\,o}{}_{;a} \tag{1.1.23}$$

> *temp23* := *expand*(*TEDS*(omega[*n*, *o*]·omega[−*o*, −*n*] = −2·omega·omega, *temp22*)) : *T*(%);

$$u_a\,\theta\,p'\omega^2-\frac{2}{3}\,\omega^2 u_a\,\theta-\frac{1}{4}\,\omega_{on}u_a u^n v^o+\frac{1}{4}\,\omega_{on}u_a u^o v^n-\frac{1}{4}\,\omega_{on}\omega^{no}{}_{;a}+\frac{1}{4}\,\omega_{no}u_a u^n v^o-\frac{1}{4}\,\omega_{no}u_a u^o v^n+\frac{1}{4}\,\omega_{no}\omega^{no}{}_{;a} \tag{1.1.24}$$

> *temp24* := *expand*(*TEDS*(omega[*n*, *o*, −*A*]·omega[−*n*, −*o*] = 2· omega·omega[−*A*], *temp23*)) : *T*(%);

$$u_a\,\theta\,p'\omega^2-\frac{2}{3}\,\omega^2 u_a\,\theta-\frac{1}{4}\,\omega_{on}u_a u^n v^o+\frac{1}{4}\,\omega_{on}u_a u^o v^n-\frac{1}{4}\,\omega_{on}\omega^{no}{}_{;a}+\frac{1}{4}\,\omega_{no}u_a u^n v^o-\frac{1}{4}\,\omega_{no}u_a u^o v^n+\frac{1}{2}\,\omega\,\omega_{;a} \tag{1.1.25}$$

> *temp25* := *expand*(*TEDS*(omega[*n*, *o*, −*A*]·omega[−*o*, −*n*] = −2· omega·omega[−*A*], *temp24*)) : *T*(%);

$$u_a\,\theta\,p'\omega^2-\frac{2}{3}\,\omega^2 u_a\,\theta-\frac{1}{4}\,\omega_{on}u_a u^n v^o+\frac{1}{4}\,\omega_{on}u_a u^o v^n+\omega\,\omega_{;a}+\frac{1}{4}\,\omega_{no}u_a u^n v^o-\frac{1}{4}\,\omega_{no}u_a u^o v^n \tag{1.1.26}$$

> *temp26* := *expand*(*TEDS*(omega[−*n*, −*o*]·*u*[*n*] = 0, *temp25*)) : *T*(%);

$$u_a\,\theta\,p'\omega^2-\frac{2}{3}\,\omega^2 u_a\,\theta-\frac{1}{4}\,\omega_{on}u_a u^n v^o+\frac{1}{4}\,\omega_{on}u_a u^o v^n+\omega\,\omega_{;a}-\frac{1}{4}\,\omega_{no}u_a u^o v^n \tag{1.1.27}$$

> *temp27* := *expand*(*TEDS*(omega[−*o*, −*n*]·*u*[*o*] = 0, *temp26*)) : *T*(%);

$$u_a\,\theta\,p'\omega^2-\frac{2}{3}\,\omega^2 u_a\,\theta-\frac{1}{4}\,\omega_{on}u_a u^n v^o+\omega\,\omega_{;a}-\frac{1}{4}\,\omega_{no}u_a u^o v^n \tag{1.1.28}$$

> *temp28* := *expand*(*TEDS*(omega[−*o*, −*n*]·*u*[*n*] = 0, *temp27*)) : *T*(%);

$$u_a\,\theta\,p'\omega^2-\frac{2}{3}\,\omega^2 u_a\,\theta+\omega\,\omega_{;a}-\frac{1}{4}\,\omega_{no}u_a u^o v^n \tag{1.1.29}$$

> *temp29* := *expand*(*TEDS*(omega[−*n*, −*o*]·*u*[*o*] = 0, *temp28*)) : *T*(%);

$$u_a\,\theta\,p'\omega^2-\frac{2}{3}\,\omega^2 u_a\,\theta+\omega\,\omega_{;a} \tag{1.1.30}$$

>

>

>

>

Hence we have shown

> *orthogvid*[22] := *P*[−*a*, *b*]·omega[*c*]·omega[−*c*, −*B*] = temp29 : *T*(%);

$$P_a{}^b\,\omega^c\,\omega_{c;b}=u_a\,\theta\,p'\omega^2-\frac{2}{3}\,\omega^2 u_a\,\theta+\omega\,\omega_{;a} \tag{1.1.31}$$

```
>
>
```

****

as a check:

```
> temp := omega[c]·omega[-c] = omega·omega : T(%);
```

$$\omega^{c}\,\omega_{c} = \omega^{2} \tag{1.1.32}$$

```
> temp2 := cod(temp,-b) : T(%);
```

$$\omega^{c}\,\omega_{c;b} + \omega^{c}{}_{;b}\,\omega_{c} = 2\,\omega\,\omega_{;b} \tag{1.1.33}$$

looking at the rhs:

```
> temp3 := 1/2 · expand(TEDS(omega[c,-B]·omega[-c] = omega[c]·omega[-c,-B],
      rhs(temp2))) : T(%);
```

$$\omega\,\omega_{;b} \tag{1.1.34}$$

```
> temp4 := P[-a,b]·temp3 : T(%);
```

$$P_{a}{}^{b}\,\omega\,\omega_{;b} \tag{1.1.35}$$

```
> temp5 := expand(TEDS(P[-a,b] = g[-a,b] + u[-a]·u[b], temp4)) : T(%);
```

$$\omega\,\omega_{;b}\,u^{b}\,u_{a} + \omega\,g_{a}{}^{b}\,\omega_{;b} \tag{1.1.36}$$

```
> temp6 := Absorbg(temp5) : T(%);
```

$$\omega\,\omega_{;b}\,u^{b}\,u_{a} + \omega\,\omega_{;a} \tag{1.1.37}$$

```
> temp7 := expand(TEDS(omega[-B]·u[b] = (`p'` - 2/3)·theta·omega, temp6)) : T(%);
```

$$u_{a}\,\theta\,p'\,\omega^{2} - \frac{2}{3}\,\omega^{2}\,u_{a}\,\theta + \omega\,\omega_{;a} \tag{1.1.38}$$

i.e.

so we have shown:

```
> HC[52] := remainder = temp7 : T(%);
```

$$P_{a}{}^{b}\,\omega^{c}\,\omega_{c;b} = u_{a}\,\theta\,p'\,\omega^{2} - \frac{2}{3}\,\omega^{2}\,u_{a}\,\theta + \omega\,\omega_{;a} \tag{1.1.39}$$

```
>
> convert(HC[52], string);
```

"P[-a,b]*omega[c]*omega[-c,-B] = u[-a]*theta*`p'`*omega^2-2/3*omega^2*u[-a]* theta+omega*omega[-A]" (1.1.40)

```
>
```